ETS PRAXIS

The *PRAXIS*® Study Companion

Elementary Education: Math Specialist (5037)

ETS Professional Educator PROGRAMS

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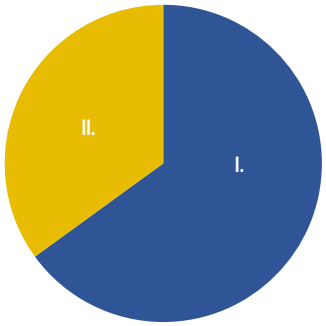
Elementary Education: Math Specialist (5037)

Test at a Glance

The *Praxis*® Elementary Education: Math Specialist test is designed to measure knowledge and competencies that are important for safe and effective beginning practice as an elementary math specialist.

Test Name	Elementary Education: Math Specialist
Test Code	5037
Time	2 hours
Number of Questions	75 selected-response and numeric-entry questions
Format	The test consists of a variety of selected-response questions, where you select one or more answer choices; questions where you enter a numeric answer in a box; and other types of questions. You can review the possible question types in Understanding Question Types.
Calculator	An on-screen four-function calculator is provided.
Test Delivery	Computer Delivered



Content Categories	Approximate Number of Questions	Approximate Percentage of Examination
I. Specialized Mathematics Knowledge for Teaching	49	65%
II. Pedagogical Knowledge and Instructional Leadership	26	35%

All questions assess content from the preceding content categories. At least 80% of questions call for application of knowledge of mathematics content.

About The Test

The math specialist is both a teacher and an instructional leader who uses a strong understanding of mathematics and mathematics instruction to support student learning, including through curriculum design and coaching to support the work and the learning of other teachers. Test takers typically have experience teaching elementary mathematics and have completed advanced preparation (i.e., a master's degree or comparable course work) in mathematics education.

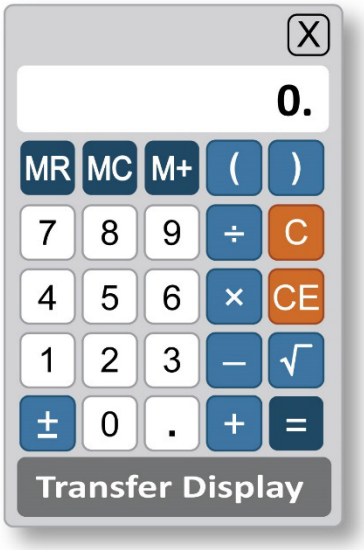
Test takers will find that at least 80% of questions call for application of knowledge of mathematics content. A full list of the mathematics topics covered is provided in Mathematics Content.

The assessment is designed and developed through work with practicing teachers and teacher educators to reflect state and national standards for the preparation of elementary math specialists. These include *Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing Programs* (2013) by the Association of Mathematics Teacher Educators (AMTE) and *NCTM CAEP Standards (2012) – Elementary Mathematics Specialist (Advanced Preparation)* by the National Council of Teachers of Mathematics (NCTM) and the Council for the Accreditation of Educator Preparation (CAEP).

This test may contain some questions that will not count toward your score.

On-Screen Four-Function Calculator

During the test, test takers have access to an on-screen four-function calculator.



Please consult the Praxis Calculator Use web page for further information and review the directions for using the on-screen calculator.

Content Topics

This list details the topics that may be included on the test. All test questions cover one or more of these topics.

Note: The use of "e.g." to start a list of examples implies that only a few examples are offered and the list is not exhaustive, whereas the use of "i.e." to start a list of examples implies that the given list of examples is complete.

Discussion Questions

In this section, discussion questions provide examples of content that may be included in the questions you receive on testing day. They are open-ended questions or statements intended to help test your knowledge of fundamental concepts and your ability to apply those concepts to classroom or real-world situations. We do **not** provide answers for the discussion questions but thinking about the answers will help improve your understanding of fundamental concepts and may help you answer a broad range of questions on the test. Most of the questions require you to combine several pieces of knowledge to formulate an integrated understanding and response. They are written to help you gain increased understanding and facility with the test's subject matter. You may want to discuss these questions with a teacher or mentor.

I. Specialized Mathematics Knowledge for Teaching

A. Instruction

1. Knows how to evaluate explanations, justifications, and definitions
   a. Identifies valid explanations of mathematical concepts (e.g., explaining why a mathematical idea is considered to be true), models, representations, or procedures
   b. Evaluates or compares explanations and justifications for their validity, generalizability, coherence, or precision, including identifying flaws in explanations and justifications
   c. Determines the changes that would improve the validity, generalizability, coherence, or precision of an explanation or justification
   d. Evaluates definitions or other mathematical language for validity, generalizability, precision, usefulness in a particular context, or support of key ideas
2. Knows how to evaluate problems, tasks, questions, examples, and procedures
   a. Identifies problems, tasks, or questions that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking

b. Identifies parallel problems that systematically vary in complexity in order to differentiate and provide appropriate challenge
c. Sequences problems, tasks, or examples based on information about learning trajectories or standards progressions
d. Evaluates the usefulness of problems, tasks, or examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
e. Identifies examples or questions that support particular strategies or address particular student questions, misconceptions, or challenges with content
f. Identifies nonexamples or counterexamples that highlight a mathematical distinction or demonstrate why a student conjecture is incorrect or partially incorrect
g. Evaluates procedures for working with mathematics content in terms of validity, appropriateness, or rigor, or to identify special cases in which the procedure might be problematic

3. Knows how to evaluate the use of representations and tools (e.g., models, manipulatives, technologies) to support a particular learning goal
a. Evaluates representations (i.e., verbal, visual, physical, contextual, symbolic) in terms of validity, generalizability, usefulness for supporting students' understanding, or fit to the concept, calculation, etc., to be represented
b. Evaluates how representations (i.e., verbal, visual, physical, contextual, symbolic) have been used to show particular ideas, relationships between ideas, processes, or strategies
c. Evaluates the appropriateness of technologies (e.g., virtual manipulatives, interactives, software) for supporting key ideas in different instructional settings (e.g., face-to-face, online, blended)

B. Student Reasoning

1. Knows how to evaluate student reasoning
a. Identifies likely misconceptions about or partial understanding of particular mathematics content and full engagement in mathematical processes and practices
b. Identifies how new mathematics content and practices can build on or connect to students' prior knowledge
c. Evaluates or compares student work (e.g., solutions, conjectures, explanations, justifications, generalizations, representations) in terms of validity, generalizability, coherence, or precision

d. Evaluates student work to identify the use of a particular concept, idea, or strategy, and purposefully sequences the presentation of student work in class discussions

e. Evaluates whether a counterargument provides an accurate critique of a given student conjecture, explanation, justification, or generalization

f. Identifies how a student's reasoning would replicate across similar problems

g. Identifies different pieces of student work that demonstrate the same or similar reasoning

C. Engagement in Mathematical Practices

1. Knows how to create, maintain, and support opportunities for learners to develop their ability to engage in mathematical practices

   a. Identifies ways to create, maintain, or support opportunities for learners to make sense of problems and persevere in solving them

   b. Identifies ways to create, maintain, or support opportunities for learners to reason abstractly and quantitatively

   c. Identifies ways to create, maintain, or support opportunities for learners to construct viable arguments and critique the reasoning of others

   d. Identifies ways to create, maintain, or support opportunities for learners to model with mathematics

   e. Identifies ways to create, maintain, or support opportunities for learners to use appropriate tools strategically

   f. Identifies ways to create, maintain, or support opportunities for learners to attend to precision

   g. Identifies ways to create, maintain, or support opportunities for learners to look for and make use of structure

   h. Identifies ways to create, maintain, or support opportunities for learners to look for and express regularity in repeated reasoning

Discussion Questions: Specialized Mathematics Knowledge for Teaching

- Identify explanations or student work samples that use specified strategies (e.g., counting on, making ten, doubles, estimation, compensation, shifting the problem, doubling and halving, composing or decomposing numbers based on place value, benchmark numbers, common denominators), properties of operations (e.g., commutative, associative, distributive), or models of operations (e.g., take-away or comparison model of subtraction, measurement or partitive model of division).

- Identify a strategy, property of operations, or model of operations used in a given explanation or student work sample.
- Identify precise mathematical definitions (e.g., prime and composite numbers, variable, categories of quadrilaterals, types of graphs that represent data).
- Identify examples of precise mathematical language or identify how to improve the precision of mathematical language (e.g., explaining a standard algorithm for multidigit addition and subtraction, using the language of place value, defining a fraction, communicating about equivalent fractions, naming fractions and decimal numbers, describing polygons).
- Identify language that remains accurate when applied to a broader set of numbers (e.g., fractions, decimals, integers). For example, recognize that greater numbers can be subtracted from lesser numbers when the set of numbers is the integers or the rational numbers.
- Identify a problem, task, or example that is aligned to a student mathematics standard.
- Identify word problems that have a given mathematical structure (e.g., compare problem with a smaller unknown, multiplicative comparison problem, division problem with an unknown quotient, partitive division problem, measurement division problem, problem represented by a given two-step linear equation).
- Identify a problem or task that would be answered using a similar process to that used in a given problem, task, or example (e.g., using one-to-one correspondence, doubling and halving, solving a linear equation in one variable).
- Identify problems that are more or less difficult than others based on standards progressions or learning trajectories (e.g., join word problem with the result unknown versus separate word problem with an initial unknown).
- Identify a concept (e.g., the meaning of the equal sign, like terms, the meaning of area) that is illustrated or introduced by a given problem, task, example, or activity.
- Identify a problem, task, example, or activity that illustrates or utilizes a given concept (e.g., commutative property, distributive property, linear functions, nonstandard units of measurement).
- Identify problems, tasks, or questions that assess students' understanding of a given concept (e.g., conservation of number, counting on, order of operations, common multiples, interpretations of remainders, place value, finding the side length of a square given its area or perimeter).

- Identify problems, tasks, examples, or actions that address a given student misconception or provide a counterexample to a given student conjecture (e.g., incomplete description of multiples, incorrect ideas about how to compare decimals or fractions, incorrect ideas about the product of fractions, incorrect ideas about the equal sign, incomplete description of a pattern represented in a table, incorrect definitions of categories of quadrilaterals).
- Identify student thinking or misconceptions that will become evident when using a given problem or task (e.g., word problems involving multidigit whole numbers, comparing fractions, measuring with a ruler).
- Identify accurate representations (e.g., models with base-10 blocks, number line models) of whole numbers, fractions, or decimals.
- Identify accurate representations (e.g., number line models, area models) of operations on whole numbers, fractions, or decimals.
- Identify statements that accurately describe representations of operations on whole numbers, fractions, or decimals.
- Identify accurate representations of mathematical terms (e.g., commutative property, measures of center).
- Identify accurate representations of specified strategies (e.g., partial sums, partial products, compensation, shifting the problem) or models of operations (e.g., take-away or comparison model of subtraction).
- Identify manipulatives (e.g., base-10 blocks) or representations (e.g., area models) that are useful for helping students understand a given concept (e.g., grouping by tens and ones, distributive property, equivalent fractions, operations with fractions).
- Identify affordances and limitations of using specified manipulatives for a given purpose (e.g., representing place value, representing fractions, naming polygons based on their attributes).
- Identify the misconception that a student most likely has based on a student's response to a given task (e.g., errors when counting, errors when interpreting remainders, errors when graphing linear equations, measurement errors, confusion about area and perimeter, misconceptions about how to classify triangles or quadrilaterals).
- Identify different content topics that are mathematically related (e.g., skip counting and multiples, metric conversions and place value).
- Identify content that should be addressed next given students' current understanding (e.g., how students classify polygons, how students solve word problems).

- Distinguish between reasoning that is valid and reasoning that is not valid when students arrive at the same answer using different methods (e.g., comparing decimals or fractions, ratio reasoning, solving linear equations).
- Identify statements that correctly evaluate the validity and/or the generalizability of a student's strategy (e.g., use of properties of operations; comparing or performing operations with whole numbers, decimals, or fractions; use of the equal sign; use of nonstandard units to measure attributes of objects).
- Identify statements that correctly characterize a student's explanation of why a claim is true (e.g., explains why the claim is true in general, assumes the claim is true, only provides examples in which the claim is true).
- Identify valid statements about mathematical content (e.g., graphs, data sets).
- Interpret students' work or explanations to determine concepts for which they demonstrate understanding, a lack of understanding, or for which the evidence is inconclusive (e.g., skip counting, cardinality, one-to-one correspondence, subitizing, conservation of number, place value concepts, measurement concepts).
- Identify the answer that a student is likely to give to a problem based on the student's answers to other problems (e.g., rounding whole numbers; performing operations with whole numbers, decimals, or fractions; finding the percent of a number; solving linear equations; finding the perimeter of a rectangle).
- Identify problems that a student is likely to answer correctly or incorrectly based on the student's answers to other problems (e.g., comparing whole numbers, decimals, or fractions; using the order of operations).
- Identify students' answers, work, or explanations that demonstrate similar reasoning (e.g., reasoning based on place value; strategies for performing operations with whole numbers, decimals, or fractions; relational thinking about the equal sign).
- Identify a problem, task, question, or action that is most closely aligned with a given mathematical practice in the context of a specified content topic.
- Identify the mathematical practice that is best represented in a given situation.

II. Pedagogical Knowledge and Instructional Leadership

A. Pedagogical Knowledge for Teaching Mathematics

1. Understands how child, preadolescent, and adult learning and development affect the mathematical learning environment
   a. Identifies ways to draw on each and every learner's mathematical strengths to create inclusive social learning contexts that engage each and every learner in discussions and mathematical explorations in order to motivate and extend learning opportunities that connect to each and every learner's experience
   b. Identifies how to demonstrate and encourage equitable and ethical treatment of each and every learner and support each and every learner in achieving high expectations
   c. Uses instructional formats (e.g., whole group, small group, partner, individual) skillfully and flexibly in support of learning goals and in consideration of various settings (e.g., face-to-face, online, blended)
   d. Identifies ways to support the equitable learning of mathematics by embracing and purposefully incorporating diversities of the classroom and school—cultural, racial, ethnic, ability, linguistic, gender, socioeconomic, developmental, etc.; uses this knowledge to motivate and extend learning opportunities
   e. Identifies ways to provide each and every learner with opportunities to make connections between mathematics and other content areas, everyday life, and the workplace
   f. Identifies ways to facilitate each and every learner's engagement in productive struggle
2. Understands equitable curriculum and assessment practices
   a. Identifies connections between mathematical concepts as well as the developmental progressions within these mathematical concepts
   b. Uses standards progressions and learning trajectories to organize and deliver instruction that is developmentally appropriate and responsive to individual learners
   c. Uses multiple strategies (e.g., asking probing questions, listening to learners) to assess mathematical knowledge and to understand thinking processes

d. Determines the suitability of mathematics curricula and teaching materials (e.g., curricular resources, technology, manipulatives) and selects, uses, and adapts those materials appropriately for particular learning goals

e. Identifies the different formats, purposes, uses, and limitations of various types of assessment of student learning in order to choose, design, or adapt assessment tasks

f. Uses the formative assessment process (administer a formative assessment task, analyze responses to the task, and determine appropriate actions based on this analysis) in order to inform teaching and benefit learning

g. Analyzes formative and summative assessment results, makes appropriate interpretations, and communicates results to appropriate and varied audiences

h. Selects and uses strategies to provide and manage timely, targeted, and effective feedback (e.g., teacher to student, student to teacher, student to student, among peers)

B. Instructional Leadership

1. Knows how to provide instructional leadership in mathematics

   a. Identifies ways to promote and support a rigorous district instructional program based on research-supported best practices regarding curriculum, instruction, technology, and assessment

   b. Selects appropriate and effective methods for communicating professionally with educational stakeholders about students, curriculum, instruction, use of technology, and assessment

   c. Demonstrates knowledge of educational structures and policies that affect equitable access to quality mathematics instruction

   d. Identifies ways to advocate for the rights and needs of each and every learner to secure resources and promote academic advancement

   e. Identifies strategies for conferring and collaborating with stakeholders to develop, implement, evaluate, and improve mathematics programs (e.g., curriculum, instruction, professional development, parent/guardian training for at-home support for students)

f. Identifies professional development needs, and selects and uses strategies to plan, develop, implement, and evaluate professional development programs at the school or district level

g. Identifies ways to use professional development (e.g., mentoring, coaching, peer-teaching, workshops) to facilitate appropriate research-supported, standards-based mathematics instruction and to promote the use of instructional methods supported by research

h. Identifies ways to support teachers in systematically reflecting and learning from practice (e.g., one-on-one observation, coaching cycle, video analysis, lesson study)

i. Applies skills and strategies for mentoring, coaching, and consultation in the development, implementation, and evaluation of an effective mathematics program

j. Identifies differences among coaching moves (e.g., telling, direct guidance, invitational guidance)

k. Identifies differences among roles on a continuum of instructional leadership (i.e., coach versus administrator)

l. Identifies ways to translate research into practices that teachers can use

m. Selects and uses strategies to determine the impact of daily and annual contributions as an elementary math specialist to mathematics teaching and learning, and uses efficacy data to advocate for the role

Discussion Questions: Pedagogical Knowledge and Instructional Leadership

- Identify equitable assessment practices.
- Identify strategies for setting goals.
- Identify strategies to provide differentiated instruction.
- Identify the affordances and limitations of different grouping strategies (e.g., heterogeneous groups, flexible groups, cooperative learning groups, whole-class instruction).
- Identify appropriate actions based on students' profiles (e.g., students' strengths or weaknesses, English learners) or information about how students respond to various interventions.
- Identify actions (e.g., teacher responses, choosing which problems to assign) that promote students' engagement in productive struggle.
- Identify examples of probing guidance.
- Identify appropriate uses of teaching materials (e.g., textbooks, manipulatives, online resources) to address a given learning goal.
- Identify primary considerations when selecting mathematics curricula.

- Define and apply the concepts of test validity, reliability, and fairness.
- Identify the purposes of, examples of, and appropriate interpretations of different types of assessments (e.g., norm-referenced tests, criterion-referenced tests, formative assessments, authentic assessments, progress monitoring, alternate assessments, diagnostic assessments).
- Identify appropriate ways to collect, interpret, and act on data to inform instructional decisions.
- Identify strategies that result in providing effective and meaningful feedback to students and teachers.
- Identify strategies to effectively build consensus and good working relationships with educational stakeholders.
- Identify strategies to effectively work with staff to implement instructional approaches or administrative directives.
- Identify actions to take personally or in collaboration with other staff members that support students' achievement in mathematics.
- Identify ways for staff (e.g., classroom teachers, special education teachers, paraprofessionals) to work together to support students.
- Apply the Response to Intervention (RTI) model to a given situation.
- Identify statements that provide appropriate reasoning for a given instructional decision or action that supports students' achievement in mathematics.
- Identify methods that effectively disseminate information to educational stakeholders (e.g., teachers, parents).
- Identify actions that facilitate effective communication and collaboration among educational stakeholders (e.g., with committees, with the larger school community).
- Identify actions and advice that support individual and corporate professional development for staff.
- Identify effective ways to structure professional development sessions and programs (e.g., lesson-study model).

Mathematics Content

Elementary Math Specialists have a deep and extensive knowledge of the foundational concepts of pre-K through 6 mathematics. Thus, an Elementary Math Specialist will not only have knowledge of the following mathematics content but will also possess deep conceptual understanding and flexible procedural understanding of these topics, understand key connections among these topics, including ways they build upon and support one another within and between grade levels, and understand how to help students and other stakeholders develop the knowledge and skills described in these topics.

Note that the percentages given for the four domains of mathematics content (i.e., Numbers and Operations, Equations and Expressions, Measurement and Geometry, Statistics) represent an approximate percentage of the questions in Categories I and II that assess mathematics content. For example, of the items in Categories I and II that assess mathematics content, 15% will be about Measurement and Geometry.

A. Numbers and Operations (60%)

1. Understands counting both conceptually and procedurally
    a. Counts and skip counts whole numbers between 0 and 1,000
    b. Counts on, starting with any whole number
    c. Connects counting to cardinality
    d. Demonstrates understanding of one-to-one correspondence between numbers and objects being counted
    e. Subitizes (recognizes small quantities by sight) conceptually and perceptually
    f. Identifies relationships between counting and the concept of larger and smaller numbers (i.e., that sets with higher counts are larger than sets with smaller counts)
2. Understands operations with whole numbers both conceptually and procedurally
    a. Demonstrates understanding of representations (i.e., verbal, visual, physical, contextual, symbolic) of addition, subtraction, multiplication, and division and connections among these representations
    b. Solves mathematical and real-world problems involving the four operations using multiple approaches and determines the reasonableness of results within the context of a given problem
    c. Uses properties of operations (e.g., commutative, associative, distributive) to solve mathematical and real-world problems in multiple ways
    d. Solves mathematical and real-world problems using basic concepts of number theory, including prime and composite numbers, factors, and multiples

3. Understands place value and decimals both conceptually and procedurally
   a. Demonstrates a conceptual understanding of the value and relationships among digits in numbers
   b. Compares multidigit and decimal numbers
   c. Compares, orders, classifies, and represents real numbers
   d. Rounds multidigit and decimal numbers
   e. Composes and decomposes multidigit numbers into groupings and explains why grouping and ungrouping are helpful in performing operations on multidigit and decimal numbers
   f. Uses drawings and objects such as manipulatives to represent place value, relating these drawings and objects to numerical equations and written descriptions
4. Understands fractions and operations with fractions both conceptually and procedurally
   a. Identifies and represents fractions as part-whole relationships, as multiples of unit fractions, as numbers, as division (e.g., $\frac{3}{4} = 3 \div 4$), and as ratios, moving back and forth flexibly among these conceptualizations
   b. Describes characteristics of fractions that are less than one, equal to one, and greater than one
   c. Partitions shapes into equal shares and recognizes that equipartitioning is a building block for understanding fractions as part-whole relationships
   d. Identifies and represents equivalent fractions
   e. Uses a variety of strategies for comparing fractions to other fractions or decimal numbers, where there are two or more numbers being compared
   f. Performs operations such as addition, subtraction, multiplication, and division with fractions as well as with fractions and whole numbers, recognizing and using different strategies for these operations, and building intuition about how the operations work (e.g., recognizing that multiplying a whole number by a fraction that is less than one makes the product smaller)
5. Understands ratios, proportions, and percents both conceptually and procedurally
   a. Applies the concepts of ratios and unit rates to describe relationships between two quantities and to solve problems
   b. Converts flexibly among equivalent decimals, fractions, and percents
   c. Identifies and represents proportional relationships
   d. Uses proportional relationships to solve ratio, percent, and scaling problems

B. Equations and Expressions (15%)

1. Understands equations and expressions both conceptually and procedurally
   a. Recognizes what it means for algebraic terms, expressions, and equations to be considered equivalent, how the equal sign is used to represent relational equivalence, and that equations maintain their equivalence status under certain algebraic manipulations
   b. Follows the standard order of operations (including the use of parentheses and the distributive property of multiplication over addition), and uses properties of operations to evaluate and manipulate algebraic expressions, equations, and formulas
   c. Uses different interpretations of the word "variable" (e.g., the ideas of quantities that are unknown, which underlies understanding of solving equations, and quantities that vary, which can be connected to patterns and will support later understanding of functional relationships) in different situations
   d. Translates between verbal statements and algebraic expressions or equations
   e. Determines whether equations are true, identifies the missing values that would make them true, solves equations using the four operations, and solves relational statements
   f. Interprets solutions of multistep one-variable linear equations and inequalities
   g. Uses linear relationships represented by equations, tables, and graphs to solve problems
   h. Uses the less-than and greater-than relational symbols ($<$, $>$) to compare quantities
   i. Uses formulas to determine unknown quantities
2. Knows how to recognize and represent patterns and functions both conceptually and procedurally
   a. Identifies, extends, describes, or generates number, shape, and contextual patterns
   b. Identifies relationships between the corresponding terms of two numerical patterns (e.g., find a rule for a function table)
   c. Determines if a function is linear or nonlinear
   d. Identifies the independent variable and dependent variable of a function
   e. Develops a function—represented by a graph, equation, or table—to model a given set of conditions
   f. Evaluates and interprets mathematical models (e.g., graph, equation, table) in context

C. Measurement and Geometry (15%)

1. Understands measurement both conceptually and procedurally
   a. Recognizes which attributes of objects are measurable and uses common measurable attributes to compare two objects
   b. Selects and uses appropriate measurement tools and standard and nonstandard units of measurement (e.g., length, area, volume, angle)
   c. Calculates and estimates perimeter, area, volume, and measurements of angles in mathematical and real-world problems, including composed shapes
   d. Uses nets that are made of rectangles and triangles to determine the surface area of three-dimensional figures
   e. Determines how changes to dimensions affect measures of area and volume
   f. Identifies relative sizes of United States customary units and metric units and converts units within each system
2. Understands geometry both conceptually and procedurally
   a. Identifies and classifies two-dimensional and three-dimensional figures and their attributes
   b. Composes and decomposes shapes
   c. Draws shapes based on specific attributes such as number of angles and number of equal faces
   d. Represents three-dimensional figures with nets, and interprets representations of two- and three-dimensional figures
   e. Uses definitions to identify lines, line segments, rays, and angles in two-dimensional figures
   f. Uses the coordinate plane and its conventions to reason and communicate about shapes and their properties and location
   g. Makes and justifies conjectures about geometric shapes and relations

D. Statistics (10%)

1. Understands basic concepts of statistics both conceptually and procedurally
   a. Identifies appropriate statistical questions and sampling procedures (e.g., randomness, size)
   b. Solves problems involving measures of center (mean, median, mode) and spread (e.g., range, variability)
   c. Determines how changes in data affect measures of center or range
   d. Describes a set of data in terms of center, spread, and shape and makes inferences from the data

2. Knows how to represent and interpret data presented in various forms
    a. Constructs and interprets graphs that represent data (e.g., bar graphs, line graphs, histograms, double bar graphs, double line graphs, boxplots, lineplots/dotplots)
    b. Chooses appropriate graphs to display data (e.g., categorical, numerical)
    c. Identifies interests that might be served by given data and how a representation of data might be misleading

Elementary Education: Math Specialist (5037) Sample Test Questions

Information about Questions That Is Specific to the Elementary Education: Math Specialist Test

- **General**
  - All numbers used are real numbers.
  - Rectangular coordinate systems are used unless otherwise stated.
  - Figures that accompany questions are intended to provide information that is useful in answering questions.
    - Figures are drawn to scale unless otherwise stated.
    - Lines shown as straight are straight, and angle measures are positive.
    - Positions of points, angles, regions, etc., exist in the order shown.
- **Types of questions that may be on the test**
  - Selected-response questions—select one answer choice
    - These are questions that ask you to select only one answer choice from a list of four choices.
  - Selected-response questions—select one or more answer choices
    - These are questions that ask you to select one or more answer choices from a list of choices. A question may or may not specify the number of choices to select. These questions are marked with square boxes beside the answer choices, not circles or ovals. See question 5 and question 13 in the Sample Test Questions.
    - If a question of this type has exactly three answer choices, one, two, or three of the choices may be correct.
    - If a question of this type has more than three answer choices, the number of correct choices will be at least 2 but fewer than the number of choices. For example, if a question of this type has five answer choices, there will be two, three, or four correct choices.
  - Selected-response questions—select an area
    - These are questions that ask you to select one or more locations on a picture or a figure (e.g., the xy-plane).

- Numeric-entry questions
  - Some of these questions ask you to enter your answer as an integer or a decimal in a single answer box. Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. See question 9 in the Sample Test Questions. Note that in these questions, the exact answer should be entered unless the question asks you to round your answer. Therefore, if one of these questions does not ask you to round your answer, you should be able to enter the exact answer in the numeric-entry box. If you are unable to do so, this may indicate that your answer is incorrect.
  - Some of these questions ask you to enter your answer as a fraction in two separate boxes—one for the numerator and one for the denominator. A negative sign can be entered in either box. Equivalent forms of the correct answer, such as $\frac{1}{2}$ and $\frac{6}{12}$, are all correct, though there may be cases in which you need to simplify your fraction so that it fits in the boxes.
- Drag-and-drop questions
  - These questions ask you to pair up given phrases or expressions by dragging (with your computer mouse) phrases or expressions from one location and matching them with given phrases or expressions in another location.
- Table grid questions
  - These questions refer to a table in which statements appear in the first column. For each statement, select the correct properties by selecting the appropriate cell(s) in the table. See question 8 in the Sample Test Questions.
- Text completion questions
  - These questions ask you to select one or more answer choices to complete one or more sentences. The choices may be located in drop-down menus in the sentences or in columns at the end of the question. You will select one answer choice from each drop-down menu or column.

Sample Questions

The sample questions that follow represent a number of the types of questions and topics that appear on the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions.

Directions: The test consists of a variety of selected-response questions, where you select one or more answer choices, and questions where you enter a numeric answer in a box.

1. A mathematics specialist is observing a teacher in a fifth-grade class. The teacher presents the following work and accompanying explanation for how to determine the difference $0.7 - 0.07$ by converting the decimals into base-10 fractions.

$$0.7 - 0.07 = \frac{7}{10} - \frac{7}{100}$$
$$= \frac{70}{100} - \frac{7}{100}$$
$$= \frac{63}{100}$$
$$= 0.63$$

First, write the decimals 7 tenths and 7 hundredths as fractions. To subtract the fractions, the denominators need to be the same, so add a zero to both the 7 and the 10 in 7 tenths. Then, 70 hundredths minus 7 hundredths is 63 hundredths, which can be written as a fraction and a decimal.

Which of the following changes to the explanation is the most important to make so that the mathematics underlying the strategy is as clear as possible?

(A) Indicating why $0.7 = \frac{7}{10}$ and $0.07 = \frac{7}{100}$

(B) Pointing out that $\frac{7 \times 10}{10 \times 10} = \frac{70}{100}$

(C) Pointing out that $\frac{70}{100} - \frac{7}{100} = \frac{70 - 7}{100}$

(D) Indicating why $0.63 = \frac{63}{100}$

2. During a unit on factors and multiples of whole numbers, Ms. Mehta wants to provide her students with a definition of a composite number.

   Which of the following definitions of a composite number is the most precise?

   (A) A whole number that is not prime
   (B) A whole number that has three or more factors
   (C) A whole number that is the product of two whole numbers
   (D) A whole number that can be divided by numbers other than itself and one

3. Mr. Conteh is selecting examples to illustrate linear functions.

   Which of the following examples best illustrates the function $y = mx$, where m is a constant?

   (A) The number of calories, y, in x ounces of an energy drink
   (B) The volume of water, y, in a tank after being drained at a constant rate for x minutes
   (C) The amount of material, y, needed for a circular parachute that has a radius of x feet
   (D) The number of bacteria, y, in a culture after x hours when the number of bacteria is increasing at a constant percent rate per hour

4. Mr. Keller's class is learning about algebraic equations. In his teacher's edition of the textbook, Mr. Keller finds a page that suggests he ask students to critique the following two solutions to determine whether they are valid.

$4x+2 = 66$	$5 = 2x+3$
$\frac{6x}{6} = \frac{66}{6}$	$\frac{5}{5} = \frac{5x}{5}$
$x = 11$	$x = 1$

   Which of the following is best addressed by the preceding task?

   (A) Misunderstanding of the properties of operations
   (B) Misunderstanding of the meaning of the equal sign
   (C) Misunderstanding of how to identify and combine like terms
   (D) Misunderstanding of how to use inverse operations to solve equations

5. At the start of a lesson on finding the side length of a square given its area, Ms. Ruffin reminded her students that a square has four sides of equal length. Then she asked them to determine the side length of a square with an area of 36 square units. Several students incorrectly answered that the side length is 9 units.

   At the end of the lesson, Ms. Ruffin wants to give a similar problem to assess whether her students are still making the same error. The students will write their final answers on slips of paper and give them to Ms. Ruffin as they exit the class.

   Which of the following area measurements would be useful for assessing student learning in this situation?

   Select **ALL** that apply.

   (A) 16 square units
   (B) 64 square units
   (C) 100 square units

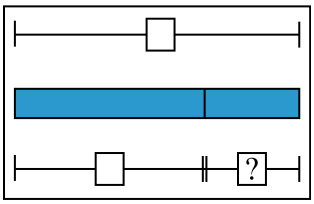
6. Ms. Howe's students are learning how to use models to help them answer word problems. The models use bars and boxes to represent the relationships between the quantities in the problems. In each model, the unknown quantity is represented with a question mark.

   Ms. Howe presents the following word problem to her students.

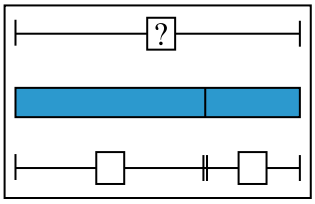
> Ruben had \$24. He gave $\frac{1}{3}$ of his money to Asha and the rest to Jada. How much money did Ruben give to Jada?

Which of the following models best represents the given word problem?

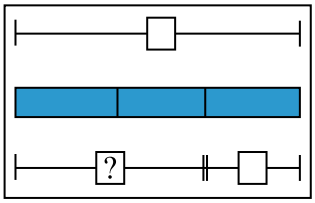
(A)



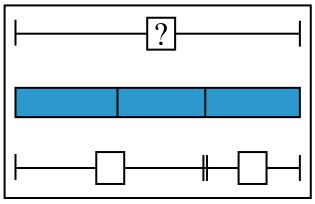
(B)



(C)



(D)



7. Sarah made a pile of 5 counters. Then Mr. Levy asked her to add counters to her pile of 5 so that the pile would have 7 counters. Sarah counted out 7 more counters and added them to the pile of 5 counters.

   Which of the following statements most likely explains the reason behind Sarah's error?

   (A) Sarah does not yet have a concept of the quantity 7.
   (B) Sarah does not yet know her number facts for sums greater than 10.
   (C) Sarah does not fully understand one-to-one correspondence between numbers and objects.
   (D) Sarah does not yet understand that one quantity can be composed of two lesser quantities.

8. Ms. Kress asked her students to compare $\frac{1}{3}$ and $\frac{7}{8}$. Four of her students correctly answered that $\frac{7}{8}$ is greater than $\frac{1}{3}$, but they gave different explanations when asked to describe their strategies to the class.

   Indicate whether each of the following student explanations provides evidence or does not provide evidence of a mathematically valid strategy for comparing $\frac{1}{3}$ and $\frac{7}{8}$.

Student Explanation	Provides Evidence	Does Not Provide Evidence
When you look at the numbers, you see that 7 is bigger than 1, so $\frac{7}{8}$ is the bigger fraction.		
In the first fraction, 1 is less than half of 3, but in the second, 7 is more than half of 8, so $\frac{7}{8}$ is larger than $\frac{1}{3}$.		
I multiplied 1 times 7 and 3 times 7, so $\frac{1}{3}$ is the same as $\frac{7}{21}$. This means that $\frac{7}{8}$ is bigger than $\frac{1}{3}$ because $\frac{1}{8}$ is bigger than $\frac{1}{21}$.		
I wanted to make a fraction equal to $\frac{1}{3}$ with the same bottom number as $\frac{7}{8}$, so I added 5 to 3 and got 8. Then I added 5 to 1 and got 6, but 7 is greater than 6, so $\frac{7}{8}$ is greater.		

9. Josh is a third-grade student in Ms. Carter's classroom. Josh's answers to three addition problems are shown in the following work.

$$\begin{array}{rrr} 385 & 453 & 321 \\ +\,462 & +\,427 & +\,836 \\ \hline 7147 & 8710 & 1157 \end{array}$$

He incorrectly answered the first two problems but correctly answered the third problem.

He will use the same strategy to answer the following problem.

$$\begin{array}{r} 328 \\ +\,564 \\ \hline \end{array}$$

What will Josh's answer be?

10. A student used the same reasoning to evaluate four expressions. The four expressions and the student's answers are given as follows. The student incorrectly evaluated the first two expressions but correctly evaluated the next two expressions.

1) $7 \times 2 - 6 + 3 = 5$
2) $9 - 5 + 16 \div 8 = 2$
3) $9 + 24 \div 3 - 1 = 16$
4) $7 \times 2 - 18 \div 6 = 11$

If the student continues to use the same reasoning, which of the following expressions is the student most likely to evaluate incorrectly?

(A) $8 + 7 - 12 \div 3$
(B) $13 - 3 \times 2 + 5$
(C) $10 \times 6 \div 15 - 3$
(D) $4 \times 5 + 10 - 12$

11. Ms. Rivera is teaching a lesson on using nonstandard units of measurement to determine the lengths of objects. She wants the lesson to engage her students in the practice of using appropriate tools strategically.

    Which of the following activities is best aligned with Ms. Rivera's goal?

    (A) Give each student a collection of paper clips of 3 sizes and a work sheet with pictures of different objects. Ask students to use their paper clips to determine the lengths of the objects.
    (B) Give each student a collection of interlocking cubes of 3 colors and a work sheet with pictures of different objects. Ask students to use their interlocking cubes to determine the lengths of the objects.
    (C) Give each student a work sheet with pictures that each consist of an object with a row of paper clips of the same size lined up next to the object. Ask students to determine the lengths of the objects.
    (D) Give each student a work sheet with pictures that each consist of an object with a row of interlocking cubes of different colors lined up next to the object. Ask students to determine the lengths of the objects.

12. Mr. Chen asked his class what 86% of 15 is and one student gave the following answer.

    "Well, I know that 86% of 15 is the same as 15% of 86. I know that 15% of 86 is 10% of 86, which is 8.6, plus 5% of 86, which is half that, so 4.3. So my answer is 12.9."

    The student's response is most connected to which of the following mathematical practices?

    (A) Modeling with mathematics
    (B) Using appropriate tools strategically
    (C) Attending to precision
    (D) Looking for and making use of structure

13. Which **THREE** of the following are the most important factors for a teacher to consider when planning differentiated instruction to maximize learning of mathematics for all students in a class?

    (A) Employing flexible grouping and regrouping practices
    (B) Using a variety of evidence-based instructional approaches to learning
    (C) Planning fast-paced instruction to spark and maintain student interest
    (D) Adapting assignments to meet the needs of a diverse student population
    (E) Assessing students' mastery of a wide range of mathematics concepts at the beginning of the year

14. At the beginning of a school year, a group of classroom teachers are reviewing assessment data from the previous year with the mathematics specialist. They notice that some students in each class scored lower than expected when answering questions involving fractions.

    Which of the following actions will best help the students during the upcoming school year?

    (A) Creating flexible groups based on the skills in which the students need support
    (B) Referring the students to the school's child-study team for further evaluation
    (C) Switching the students to classrooms with teachers who are comfortable teaching fractions
    (D) Providing the students with individual tutoring for each fraction concept with which they had difficulty

15. Ms. Westcott wants to assess her students' understanding of different classifications of triangles in a way that facilitates their engagement in productive struggle.

    Which of the following tasks is most likely to meet Ms. Westcott's goal?

    (A) Ask students to write the definition of each of the following classifications of triangles: acute, right, obtuse, scalene, isosceles, and equilateral.
    (B) Provide students with pictures of 7 triangles, and have them classify the triangles as acute, right, or obtuse, and scalene, isosceles, or equilateral.
    (C) Give students the measures of the three angles for each of 7 triangles, and have them classify the triangles as acute, right, or obtuse, and scalene, isosceles, or equilateral.
    (D) Tell students that one angle in a triangle measures 80 degrees, and have them explain whether the triangle could be acute, right, obtuse, scalene, isosceles, or equilateral.

16. A curriculum development team at an elementary school will be conducting an in-depth analysis and revamping of the school's mathematics curriculum.

    During the planning and development phase of the project, which of the following steps is the best for the team to take first?

    (A) Developing a multigrade scope and sequence document and a curriculum map
    (B) Analyzing state and national standards to formulate a philosophy and rationale for a revised curriculum
    (C) Recruiting trainers and mentors responsible for professional development sessions using new curriculum materials
    (D) Reviewing local, state, and national test data and stakeholder survey results to guide the selection of a curriculum

17. Mr. Dominguez is teaching a unit on representing data with bar graphs, picture graphs, and line plots and on solving problems using information presented in these graphs.

    The most effective formative evaluation of the success of the unit is primarily dependent on which of the following?

    (A) A comparison of students' pretest and posttest scores on a criterion-referenced assessment
    (B) A tightly controlled research design that permits data-based decisions
    (C) A normed diagnostic assessment for making instructional decisions
    (D) Continuous monitoring and adjustment as feedback indicates

18. Following a meeting with fourth-grade teachers, a mathematics specialist realizes that some of the problems students have in developing a conceptual understanding of place value and using this understanding to perform operations with whole numbers are the result of the inconsistency of mathematics instruction throughout the school.

    To begin to solve the problem, which of the following actions should the mathematics specialist take first?

    (A) Discuss the problem with the third-grade teachers and ask them for recommendations for change.
    (B) Select a new mathematics curriculum that has been used successfully in other schools and require its implementation.
    (C) Advise the fourth-grade teachers to spend the next six weeks of instruction reteaching place value concepts to students.
    (D) Meet with teachers to form a mathematics committee to explore ways to improve students' understanding of place value concepts.

19. Which of the following is the best way for an elementary mathematics specialist who plans and facilitates professional learning sessions to collaborate with a group of teachers?

    (A) Observing lessons and writing evaluations for teachers
    (B) Selecting professional literature that teachers must read
    (C) Structuring meetings during which teachers focus on goals
    (D) Encouraging the principal to meet with teachers resistant to change

Answers

1. Option (B) is correct. When the teacher tried to explain why $\frac{7}{10} = \frac{70}{100}$, what the teacher really said was that $\frac{7+0}{10+0} = \frac{70}{100}$. However, 70 comes from multiplying 7 by 10, and 100 comes from multiplying 10 by 10, so a better explanation would be to say that it is necessary to multiply the numerator and denominator of $\frac{7}{10}$ by 10 to obtain the equivalent fraction $\frac{70}{100}$. Note that the teacher clearly conveyed that $0.7 = \frac{7}{10}$ and $0.07 = \frac{7}{100}$ by referring to the decimals 0.7 and 0.07 as "7 tenths" and "7 hundredths," respectively, which linked each of the two decimal numbers to its corresponding base-10 fraction. Also, the teacher pointed out that $\frac{70}{100} - \frac{7}{100} = \frac{70-7}{100}$, by saying, "Then, 70 hundredths minus 7 hundredths is 63 hundredths." Finally, the teacher stated that the number 63 hundredths can be written as the base-10 fraction $\frac{63}{100}$ and the decimal 0.63.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	A. Instruction
Topic	1. Knows how to evaluate explanations, justifications, and definitions
Subtopic	c. Determines the changes that would improve the validity, generalizability, coherence, or precision of an explanation or justification
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	3. Understands place value and decimals both conceptually and procedurally
Mathematics Content Subtopic	e. Composes and decomposes multidigit numbers into groupings and explains why grouping and ungrouping are helpful in performing operations on multidigit and decimal numbers

2. Option (B) is correct. This definition is the only one of the options given that contains the idea that composite numbers are whole numbers that have more than two factors. The whole numbers 0 and 1 satisfy the definition in option (A) because they are not prime, but they are not composite either. The definition in option (C) describes all whole numbers, not just composite numbers. The definition in option (D) also describes all whole numbers, since any whole number can be divided by nonzero numbers (including fractions and decimals) other than itself and one.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	A. Instruction
Topic	1. Knows how to evaluate explanations, justifications, and definitions
Subtopic	d. Evaluates definitions or other mathematical language for validity, generalizability, precision, usefulness in a particular context, or support of key ideas
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	2. Understands operations with whole numbers both conceptually and procedurally
Mathematics Content Subtopic	d. Solves mathematical and real-world problems using basic concepts of number theory, including prime and composite numbers, factors, and multiples

3. Option (A) is correct. Since the function $y = mx$ has a linear term but no constant term, an example that illustrates the function needs to be a proportional relationship. Since the number of calories consumed is proportional to the number of ounces of the energy drink that are consumed, the example in option (A) illustrates the function $y = mx$. Option (B) is an example of a linear function, but it contains a constant term since there is an initial amount of water in the pool, so this example does not illustrate the function $y = mx$. Option (C) is an example of a quadratic function, and option (D) is an example of an exponential function.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	A. Instruction
Topic	2. Knows how to evaluate problems, tasks, questions, examples, and procedures
Subtopic	d. Evaluates the usefulness of problems, tasks, or examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
Mathematics Content Domain	B. Equations and Expressions
Mathematics Content Topic	2. Knows how to recognize and represent patterns and functions both conceptually and procedurally
Mathematics Content Subtopic	c. Determines if a function is linear or nonlinear

4. Option (C) is correct. In the first solution, $4x$ and 2 are added to get $6x$, but the $4x$ term contains a variable, whereas the 2 is a constant term; it is incorrect to add $4x$ and 2 because they are not like terms. Similarly, in the second solution, $2x$ and 3 are added to get $5x$, but $2x$ and 3 are not like terms, so this strategy is not valid. Therefore, a misunderstanding of how to identify and combine like terms is the option that is best addressed by asking students to critique the two invalid strategies.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	A. Instruction
Topic	2. Knows how to evaluate problems, tasks, questions, examples, and procedures
Subtopic	e. Identifies examples or questions that support particular strategies or address particular student questions, misconceptions, or challenges with content
Mathematics Content Domain	B. Equations and Expressions
Mathematics Content Topic	1. Understands equations and expressions both conceptually and procedurally
Mathematics Content Subtopic	e. Determines whether equations are true, identifies the missing values that would make them true, solves equations using the four operations, and solves relational statements

5. Options (B) and (C) are correct. At the start of the lesson, several students answered that the side length of a square with an area of 36 square units is 9 units instead of giving the correct side length of 6 units. Since the side length of a square with a perimeter of 36 units is 9 units, the students are probably confusing area and perimeter. Therefore, an area that would **NOT** be useful for assessing student learning in this situation is one that would allow a student to find the correct answer by dividing the number of square units by 4, since the perimeter of a square is divided by 4 to find the side length of the square. In both (B) and (C), the square root of the number of square units is not equal to the result when the number of square units is divided by 4, so these areas would be useful for assessing student learning in this situation. However, in (A), $\sqrt{16} = 4$ and $16 \div 4 = 4$. Since the answers are the same, Ms. Ruffin would have no way of knowing whether students were thinking about area or thinking about perimeter when finding the answer, so the problem in (A) is not useful for assessing student learning in this situation.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	A. Instruction
Topic	2. Knows how to evaluate problems, tasks, questions, examples, and procedures
Subtopic	e. Identifies examples or questions that support particular strategies or address particular student questions, misconceptions, or challenges with content
Mathematics Content Domain	C. Measurement and Geometry
Mathematics Content Topic	1. Understands measurement both conceptually and procedurally
Mathematics Content Subtopic	c. Calculates and estimates perimeter, area, volume, and measurements of angles in mathematical and real-world problems, including composed shapes

6. Option (C) is correct. First, since it is known that Ruben had $24, the model should show that the total amount is known. Next, since Ruben gave $\frac{1}{3}$ of his money to Asha, the model should show that the total is divided in thirds. Finally, the model should show that the money given to Jada, which is $\frac{2}{3}$ of the total amount, is the unknown. Option (C) is the only option that shows that the total amount would be filled in, the total amount is divided in thirds, and the unknown is $\frac{2}{3}$ of the total amount. Note that option (A) shows that the unknown is the lesser part of the whole, and options (B) and (D) both show that the unknown is the total amount. Also, in both options (A) and (B), all that is given is that one part is larger than the other, not that one part is $\frac{2}{3}$ of the whole and the other part is $\frac{1}{3}$ of the whole.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	A. Instruction
Topic	3. Knows how to evaluate the use of representations and tools (e.g., models, manipulatives, technologies) to support a particular learning goal
Subtopic	a. Evaluates representations (i.e., verbal, visual, physical, contextual, symbolic) in terms of validity, generalizability, usefulness for supporting students' understanding, or fit to the concept, calculation, etc., to be represented
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	4. Understands fractions and operations with fractions both conceptually and procedurally
Mathematics Content Subtopic	f. Performs operations such as addition, subtraction, multiplication, and division with fractions as well as with fractions and whole numbers, recognizing and using different strategies for these operations, and building intuition about how the operations work (e.g., recognizing that multiplying a whole number by a fraction that is less than one makes the product smaller)

7. Option (D) is correct. Sarah counted out 7 more counters, not realizing that 5 can be part of 7, so she does not seem to understand that one quantity can be composed of two lesser quantities. Note that since Sarah counted out 7 counters, there is evidence that she has a concept of the quantity 7 and there is evidence that she understands one-to-one correspondence. Also, since Sarah's task was to add counters to her pile of 5 counters so there would be 7 counters in the pile, it was not necessary for her to know number facts for sums greater than 10 to complete the task.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	B. Student Reasoning
Topic	1. Knows how to evaluate student reasoning
Subtopic	a. Identifies likely misconceptions about or partial understanding of particular mathematics content and full engagement in mathematical processes and practices
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	1. Understands counting both conceptually and procedurally
Mathematics Content Subtopic	b. Counts on, starting with any whole number

8. The first and fourth explanations do not provide evidence of a mathematically valid strategy for comparing $\frac{1}{3}$ and $\frac{7}{8}$, but the second and third explanations do. In the first explanation, the student compares only the numerators of the fractions, which is not a valid strategy because it does not take into account the effect of the denominator on the size of the pieces. In the second explanation, the student compares both fractions to the benchmark fraction $\frac{1}{2}$, which is a valid strategy since $\frac{1}{3}$ is less than $\frac{1}{2}$ and $\frac{7}{8}$ is greater than $\frac{1}{2}$. In the third explanation, the student uses multiplicative reasoning to find a common numerator, and then the student compares the fractions by reasoning about the sizes of the unit fractions $\frac{1}{8}$ and $\frac{1}{21}$. This is a valid strategy. In the fourth explanation, the student uses additive reasoning to try to find a fraction equivalent to $\frac{1}{3}$ that has a denominator of 8, but $\frac{6}{8}$ is not equivalent to $\frac{1}{3}$, so this strategy is not valid.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	B. Student Reasoning
Topic	1. Knows how to evaluate student reasoning
Subtopic	c. Evaluates or compares student work (e.g., solutions, conjectures, explanations, justifications, generalizations, representations) in terms of validity, generalizability, coherence, or precision
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	4. Understands fractions and operations with fractions both conceptually and procedurally
Mathematics Content Subtopic	e. Uses a variety of strategies for comparing fractions to other fractions or decimal numbers, where there are two or more numbers being compared

9. The correct answer is 8812. Josh's error is that he is not regrouping when necessary; instead he is just writing the sum of the digits in each place value column. His written answer is correct in the third problem because 11 hundreds (the result of adding 3 hundreds and 8 hundreds) is equivalent to regrouping to get 1100. However, when he does not regroup in the first two problems, his written answers are incorrect. For example, in the first problem, Josh adds 8 tens and 6 tens to get 14 tens, but instead of regrouping 10 of those tens to get 100 and then writing the final answer as 847, Josh just adds the 3 hundreds and the 4 hundreds and then writes the final answer as 7147. Therefore, if Josh uses the same method in the last problem, he will add 8 and 4 to get 12 ones, but he will not regroup, and then he will add 2 and 6 to get 8 and 3 and 5 to get 8, and his final answer will be 8812.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	B. Student Reasoning
Topic	1. Knows how to evaluate student reasoning
Subtopic	f. Identifies how a student's reasoning would replicate across similar problems
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	2. Understands operations with whole numbers both conceptually and procedurally
Mathematics Content Subtopic	b. Solves mathematical and real-world problems involving the four operations using multiple approaches and determines the reasonableness of results within the context of a given problem

10. Option (B) is correct. A common misconception about the order of operations is that multiplication always comes before division and that addition always comes before subtraction. In the first expression, the student evaluated $7 \times 2 - (6 + 3)$ instead of $7 \times 2 - 6 + 3$, and in the second expression, the student evaluated $9 - (5 + 16 \div 8)$ instead of $9 - 5 + 16 \div 8$. In the next two expressions, the misconception described does not interfere with a student's ability to correctly evaluate the expressions, and the student obtained the correct answers, so it is likely that this misconception is the basis for the student's incorrect answers. In the expressions in (A), (C), and (D), this misconception does not interfere with a student's ability to correctly evaluate the expressions either, but in the expression in (B), it is incorrect to add before subtracting, so the expression in (B) is the one that the student is most likely to evaluate incorrectly.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	B. Student Reasoning
Topic	1. Knows how to evaluate student reasoning
Subtopic	f. Identifies how a student's reasoning would replicate across similar problems
Mathematics Content Domain	B. Equations and Expressions
Mathematics Content Topic	1. Understands equations and expressions both conceptually and procedurally
Mathematics Content Subtopic	b. Follows the standard order of operations (including the use of parentheses and the distributive property of multiplication over addition), and uses properties of operations to evaluate and manipulate algebraic expressions, equations, and formulas

11. Option (A) is correct. Option (A) is the only activity where students are provided with tools of different lengths (i.e., the paper clips of different sizes), and they need to determine which tool to use and how to use the tool correctly to determine the lengths of the objects (e.g., measuring with no gaps or overlaps). In all of the other activities, students are given tools of the same size to use, so the choice has already been made for them.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	C. Engagement in Mathematical Practices
Topic	1. Knows how to create, maintain, and support opportunities for learners to develop their ability to engage in mathematical practices
Subtopic	e. Identifies ways to create, maintain, or support opportunities for learners to use appropriate tools strategically
Mathematics Content Domain	C. Measurement and Geometry
Mathematics Content Topic	1. Understands measurement both conceptually and procedurally
Mathematics Content Subtopic	b. Selects and uses appropriate measurement tools and standard and nonstandard units of measurement (e.g., length, area, volume, angle)

12. Option (D) is correct. The student's approach to the problem demonstrated a deep understanding of the mathematical structure of the calculation of a percent of a number, which led to a more efficient way of solving the problem.

Content	I. Specialized Mathematics Knowledge for Teaching
Subcategory	C. Engagement in Mathematical Practices
Topic	1. Knows how to create, maintain, and support opportunities for learners to develop their ability to engage in mathematical practices
Subtopic	g. Identifies ways to create, maintain, or support opportunities for learners to look for and make use of structure
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	5. Understands ratios, proportions, and percents both conceptually and procedurally
Mathematics Content Subtopic	d. Uses proportional relationships to solve ratio, percent, and scaling problems

13. Options (A), (B), and (D) are correct. Option (A) is correct because it is best practice for a teacher to use a combination of whole-class, small-group, and individual instruction when planning differentiated instruction and to change groupings based on students' learning needs. Option (B) is correct because when providing differentiated instruction, it is important for a teacher to use multiple strategies that are selected to best meet specific needs. Option (D) is correct because it is best practice for a teacher to adapt students' assignments based on their individual learning needs. Note that student interest is not always dependent on delivering instruction at a fast pace, and it is best practice to assess students on a more narrow range of concepts on an ongoing basis so that instruction can better meet students' individual learning needs.

Content	II. Pedagogical Knowledge and Instructional Leadership
Subcategory	A. Pedagogical Knowledge for Teaching Mathematics
Topic	1. Understands how child, preadolescent, and adult learning and development affect the mathematical learning environment
Subtopic	b. Identifies how to demonstrate and encourage equitable and ethical treatment of each and every learner and support each and every learner in achieving high expectations

14. Option (A) is correct. After reviewing the data, teachers can adjust small-group instruction based on students' needs and periodically modify the groups depending on student learning. Note that further evaluation by a school's child-study team is not automatically required when students need support with some skills, moving the students to the classrooms of teachers who are comfortable teaching fractions may not improve the students' skills if remediation is not provided, and individually tutoring students on multiple topics is time-consuming and inefficient.

Content	II. Pedagogical Knowledge and Instructional Leadership
Subcategory	A. Pedagogical Knowledge for Teaching Mathematics
Topic	1. Understands how child, preadolescent, and adult learning and development affect the mathematical learning environment
Subtopic	c. Uses instructional formats (e.g., whole group, small group, partner, individual) skillfully and flexibly in support of learning goals and in consideration of various settings (e.g., face-to-face, online, blended)
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	4. Understands fractions and operations with fractions both conceptually and procedurally
Mathematics Content Subtopic	a. Identifies and represents fractions as part-whole relationships, as multiples of unit fractions, as numbers, as division (e.g., $\frac{3}{4} = 3 \div 4$), and as ratios, moving back and forth flexibly among these conceptualizations

15. Option (D) is correct. In the task in option (D), students need to consider how the relationships among the angles of the triangle change as the measures of the angles change. This is more likely to lead to productive struggle than the other tasks, which focus on classifying well-defined triangles or just defining mathematical terms.

Content	II. Pedagogical Knowledge and Instructional Leadership
Subcategory	A. Pedagogical Knowledge for Teaching Mathematics
Topic	1. Understands how child, preadolescent, and adult learning and development affect the mathematical learning environment
Subtopic	f. Identifies ways to facilitate each and every learner's engagement in productive struggle
Mathematics Content Domain	C. Measurement and Geometry
Mathematics Content Topic	2. Understands geometry both conceptually and procedurally
Mathematics Content Subtopic	a. Identifies and classifies two-dimensional and three-dimensional figures and their attributes

16. Option (B) is correct. The review and revision of a mathematics curriculum begins by analyzing state and national standards. Focusing on establishing standards for what students should learn is a first step in creating the philosophy and rationale that will guide the development of a revised curriculum. Note that the step in option (B) should precede the step in option (D), the step in option (D) should precede the step in option (A), and the step in option (A) should precede the step in option (C).

Content	II. Pedagogical Knowledge and Instructional Leadership
Subcategory	A. Pedagogical Knowledge for Teaching Mathematics
Topic	2. Understands equitable curriculum and assessment practices
Subtopic	d. Determines the suitability of mathematics curricula and teaching materials (e.g., curricular resources, technology, manipulatives) and selects, uses, and adapts those materials appropriately for particular learning goals

17. Option (D) is correct. Formative evaluation of the unit involves assessing, monitoring, and analyzing data on an ongoing basis, and the only option that highlights ongoing evaluation is option (D).

Content	II. Pedagogical Knowledge and Instructional Leadership
Subcategory	A. Pedagogical Knowledge for Teaching Mathematics
Topic	2. Understands equitable curriculum and assessment practices
Subtopic	f. Uses the formative assessment process (administer a formative assessment task, analyze responses to the task, and determine appropriate actions based on this analysis) in order to inform teaching and benefit learning
Mathematics Content Domain	D. Statistics
Mathematics Content Topic	2. Knows how to represent and interpret data presented in various forms
Mathematics Content Subtopic	a. Constructs and interprets graphs that represent data (e.g., bar graphs, line graphs, histograms, double bar graphs, double line graphs, boxplots, lineplots/dotplots)

18. Option (D) is correct. To best make change in a school, ideas for reform should come from within and teachers should have ownership. Collaboration between the stakeholders in the school is most likely to result in the most effective change.

Content	II. Pedagogical Knowledge and Instructional Leadership
Subcategory	B. Instructional Leadership
Topic	1. Knows how to provide instructional leadership in mathematics
Subtopic	e. Identifies strategies for conferring and collaborating with stakeholders to develop, implement, evaluate, and improve mathematics programs (e.g., curriculum, instruction, professional development, parent/guardian training for at-home support for students)
Mathematics Content Domain	A. Numbers and Operations
Mathematics Content Topic	3. Understands place value and decimals both conceptually and procedurally
Mathematics Content Subtopic	a. Demonstrates a conceptual understanding of the value and relationships among digits in numbers

19. Option (C) is correct. The elementary mathematics specialist working collaboratively with teachers provides structure by helping the teachers focus on goals. Note that the actions described in options (A), (B), and (D) are not collaborative in nature.

Content	II. Pedagogical Knowledge and Instructional Leadership
Subcategory	B. Instructional Leadership
Topic	1. Knows how to provide instructional leadership in mathematics
Subtopic	f. Identifies professional development needs, and selects and uses strategies to plan, develop, implement, and evaluate professional development programs at the school or district level

Understanding Question Types

The *Praxis*® assessments include a variety of question types: constructed response (for which you write a response of your own); selected response, for which you select one or more answers from a list of choices or make another kind of selection (e.g., by selecting a sentence in a text or by selecting part of a graphic); and numeric entry, for which you enter a numeric value in an answer field. You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don't spend time during the test figuring out how to answer them.

Understanding Selected-Response and Numeric-Entry Questions

For most questions, you respond by selecting an oval to select a single answer from a list of answer choices.

However, interactive question types may also ask you to respond by:

- Selecting more than one choice from a list of choices.
- Typing in a numeric-entry box. When the answer is a number, you may be asked to enter a numerical answer. Some questions may have more than one entry box to enter a response. Numeric-entry questions typically appear on mathematics-related tests.
- Selecting parts of a graphic. In some questions, you will select your answers by selecting a location (or locations) on a graphic such as a map or chart, as opposed to choosing your answer from a list.
- Selecting sentences. In questions with reading passages, you may be asked to choose your answers by selecting a sentence (or sentences) within the reading passage.
- Dragging and dropping answer choices into targets on the screen. You may be asked to select answers from a list of choices and to drag your answers to the appropriate location in a table, paragraph of text or graphic.
- Selecting answer choices from a drop-down menu. You may be asked to choose answers by selecting choices from a drop-down menu (e.g., to complete a sentence).

Remember that with every question you will get clear instructions.

Understanding Constructed-Response Questions

Some tests include constructed-response questions, which require you to demonstrate your knowledge in a subject area by writing your own response to topics. Essays and short-answer questions are types of constructed-response questions.

For example, an essay question might present you with a topic and ask you to discuss the extent to which you agree or disagree with the opinion stated. You must support your position with specific reasons and examples from your own experience, observations, or reading.

Review a few sample essay topics:

- *Brown v. Board of Education of Topeka*

  "We come then to the question presented: Does segregation of children in public schools solely on the basis of race, even though the physical facilities and other 'tangible' factors may be equal, deprive the children of the minority group of equal educational opportunities? We believe that it does."

  A. What legal doctrine or principle, established in *Plessy v. Ferguson* (1896), did the Supreme Court reverse when it issued the 1954 ruling quoted above?
  B. What was the rationale given by the justices for their 1954 ruling?

- *In his self-analysis, Mr. Payton says that the better-performing students say small-group work is boring and that they learn more working alone or only with students like themselves. Assume that Mr. Payton wants to continue using cooperative learning groups because he believes they have value for all students.*

  - Describe **TWO** strategies he could use to address the concerns of the students who have complained.
  - Explain how each strategy suggested could provide an opportunity to improve the functioning of cooperative learning groups. Base your response on principles of effective instructional strategies.

- *"Minimum-wage jobs are a ticket to nowhere. They are boring and repetitive and teach employees little or nothing of value. Minimum-wage employers take advantage of people because they need a job."*

  - Discuss the extent to which you agree or disagree with this opinion. Support your views with specific reasons and examples from your own experience, observations, or reading.

Keep these things in mind when you respond to a constructed-response question:

1. **Answer the question accurately.** Analyze what each part of the question is asking you to do. If the question asks you to describe or discuss, you should provide more than just a list.
2. **Answer the question completely.** If a question asks you to do three distinct things in your response, you should cover all three things for the best score. Otherwise, no matter how well you write, you will not be awarded full credit.
3. **Answer the question that is asked.** Do not change the question or challenge the basis of the question. You will receive no credit or a low score if you answer another question or if you state, for example, that there is no possible answer.
4. **Give a thorough and detailed response.** You must demonstrate that you have a thorough understanding of the subject matter. However, your response should be straightforward and not filled with unnecessary information.
5. **Take notes on scratch paper** so that you don't miss any details. Then you'll be sure to have all the information you need to answer the question.
6. **Reread your response.** Check that you have written what you thought you wrote. Be sure not to leave sentences unfinished or omit clarifying information.

General Assistance For The Test

Praxis® Interactive Practice Test

This full-length *Praxis*® practice test lets you practice answering one set of authentic test questions in an environment that simulates the computer-delivered test.

- Timed just like the real test
- Correct answers with detailed explanations
- Practice test results for each content category

ETS provides a free interactive practice test with each test registration. You can learn more here.

Doing Your Best

Strategy and Success Tips

Effective *Praxis* test preparation doesn't just happen. You'll want to set clear goals and deadlines for yourself along the way. Learn from the experts. Get practical tips to help you navigate your Praxis test and make the best use of your time. Learn more at Strategy and Tips for Taking a *Praxis* Test.

Develop Your Study Plan

Planning your study time is important to help ensure that you review all content areas covered on the test. View a sample plan and learn how to create your own. Learn more at Develop a Study Plan.

Helpful Links

Ready to Register – How to register and the information you need to know to do so.

Disability Accommodations – Testing accommodations are available for test takers who meet ETS requirements.

PLNE Accommodations (ESL) – If English is not your primary language, you may be eligible for extended testing time.

What To Expect on Test Day – Knowing what to expect on test day can make you feel more at ease.

Getting Your Scores – Find out where and when you will receive your test scores.

State Requirements – Learn which tests your state requires you to take.

Other Praxis Tests – Learn about other *Praxis* tests and how to prepare for them.

To search for the *Praxis* test prep resources that meet your specific needs, visit:

www.ets.org/praxis/testprep

To purchase official test prep made by the creators of the *Praxis* tests, visit the ETS Store:

www.ets.org/praxis/store

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