MASSACHUSETTS
TESTS FOR EDUCATOR LICENSURE®

Practice Test:

Mathematics (63)

Answer Key, Sample Responses, Evaluation Chart, and Score Calculation Tool

Answer Key

When you have finished the practice test, click on "show answers" to see how well you did on each objective. In addition, use the Evaluation Chart to determine how many questions within each objective you answered correctly.

You will not receive a score for the practice test, and there is no passing score for the practice test. However, to get a sense of how well you did, use the Score Calculation Tool to better gauge your performance and degree of readiness to take an MTEL test at an operational administration.

NOTE: When you take the actual test, you will receive a score report that provides subarea-level performance, not objective-level performance. Information about test results can be found at Score Report Explanation (../../PageView.aspx?f=GEN_UnderstandingYourTestResults.html).

Question Number	Your Response	Correct Response	Related Objectives and Rationale
1		B	Objective 001 **Correct Response: B.** Let p = the price the owner paid for a shirt. Then $1.2p$ is the amount the first shirt is sold for. The cost of 2 shirts during the sale is $1.2p + \frac{1.2p}{2} = 1.8p$. Thus, $1.8p \div 2 = 0.9p$ is the sale price of one shirt. Since the owner paid p for the shirt, $p - 0.9p = 0.1p$ = 10% of p is the loss on each shirt. **Incorrect Response: A.**This response could be the result of thinking there is a 20% profit on the first shirt bought on sale and a 10% profit on the second shirt. This is incorrect because both the cost of the second shirt and its 20% markup must be halved to determine its selling price. By this line of reasoning, there is an average profit of 15% for each shirt—a difference of 20% – 15% = 5%. **Incorrect Response: C.**This response could be the result of forgetting to divide $1.8p$ by 2 and thinking that $1.8p$ is the sale price for 2 shirts, so $2p - 1.8p = 0.2p$ = a 20% loss. **Incorrect Response: D.**This response could be the result of thinking that a customer pays 100% of the retail price for the first shirt, 50% of the retail price for the second shirt, resulting in (100% + 50%) ÷ 2 = 75% of the retail price per shirt, so the owner loses 25% per shirt.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
2		B	Objective 001 **Correct Response: B.** Substitute the values given for ρ, ℓ, and A: $R = \frac{(1.75\times10^{-6})(2.5\times10^{2})}{3.5\times10^{-3}} = \left(\frac{1.75\times2.5}{3.5}\right)\times10^{-6+2-(-3)} = 1.25 \times 10^{-1} = 0.125.$ **Incorrect Response: A.** This response could be the result of attempting to perform the calculation shown in the correct response but then incorrectly subtracting the –3 exponent: –6 + 2 – 3 = –7. **Incorrect Response: C.** This response could be the result of incorrectly interpreting 0.125 as 1.25×10^1. **Incorrect Response: D.** This response could be the result of multiplying and dividing the exponents, (–6)(2) ÷ (–3), rather than adding and subtracting them.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
3		A	Objective 001 **Correct Response: A.** Since all arrangements are identical and they all are used, the number of arrangements is the greatest common factor of the number of flowers. There are 192 roses (16 × 12), 160 lilies, and 128 carnations. 192 = $2^6 \times 3$, 160 = $2^5 \times 3$, and 128 = 2^7. The greatest common factor is $2^5 = 32$ arrangements. There are 480 flowers, so there are 480 ÷ 32 = 15 flowers in each arrangement. **Incorrect Response: B.** This response could be the result of forgetting to multiply 16 by 12 (i.e., there are 16 dozen roses), then responding with the number of arrangements rather than the number of flowers in each. The greatest common factor of 16, 160, and 128 is 16. Thus, this error would give 16 flowers per arrangement. **Incorrect Response: C.** This response could be the result of calculating the number of arrangements as 16 as in response B, then dividing the number of flowers, 480, by 16 to get 30 flowers per arrangement. **Incorrect Response: D.** This response is the number of arrangements, not the number of flowers in each arrangement.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
4		C	Objective 001 **Correct Response: C.** The relationships that appear in the computation may be restated with multiplication as $d(cd) + b = bae$, where each letter represents a digit (i.e., *cd* represents a 2-digit number and *bae* represents a 3-digit number). Divide both sides of the equation by *d* to obtain the sum $cd + \frac{b}{d}$ represents the product of digits *c* and *d* plus the fraction $\frac{b}{d}$. This may be expressed as the mixed number $cd\frac{b}{d}$. **Incorrect Response: A.** This equation misinterprets the value of *b* to represent "*b* tenths" of a unit. This would only be true when the divisor is 10, and this cannot be true for the work shown because divisor *d* must be a digit from 1 to 9. **Incorrect Response: B.** This is incorrect because the remainder must also be divided by *d*. As an example of why this is incorrect, let *bae* be the number 126 and *d* be the number 5. The equation shown then implies that $(126 \div 5) = 25 + 1 = 26$. However, $25 \times 5 = 125$ and $26 \times 5 = 130$. The correct division is $(126 \div 5) = (125 + 1) \div 5 = 25 + 1 \div 5$. **Incorrect Response: D.** This equation shows the remainder, *b*, being divided by the dividend, *bae*. This is incorrect. The remainder must also be divided by the divisor.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
5		C	Objective 001 **Correct Response: C.** The area of the rectangle is equal to $\sqrt{3} \cdot (\sqrt{3} + \sqrt{5})$. By the distributive property, this becomes $(\sqrt{9} + \sqrt{15}) \approx 3 + 3.9 \approx 7$. **Incorrect Response: A.** This response may reflect a misapplication of the distributive property: $\sqrt{3} \cdot \sqrt{3} + \sqrt{5} \approx 3 + 2.2 \approx 5$. It may also reflect the misconception that $\sqrt{3} + \sqrt{5} = \sqrt{8}$, which leads to the conclusion that $\sqrt{3} \cdot (\sqrt{3} + \sqrt{5}) = \sqrt{3} \cdot \sqrt{8} = \sqrt{24} \approx 5$. **Incorrect Response: B.** This response may be the result of interpreting a square root to mean "half" (e.g., $\sqrt{3} = \frac{3}{2}$ and $\sqrt{5} = \frac{5}{2}$): $\frac{3}{2}\left(\frac{3}{2} + \frac{5}{2}\right) = \frac{3}{2}(4) = 6$. **Incorrect Response: D.** This response may be the result of an estimation error. If $\sqrt{3}$ and $\sqrt{5}$ are both estimated to be 2 (perhaps by reasoning that an overestimation from approximating $\sqrt{3}$ as 2 is counterbalanced by an underestimation from approximating $\sqrt{5}$ as 2), then the calculation would be $2(2 + 2) = 8$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
6		C	Objective 001 **Correct Response: C.** The radical expressions may be separated and written as $\frac{\sqrt[3]{27}\cdot\sqrt[3]{x}\cdot\sqrt[4]{16}\cdot\sqrt[4]{x}}{\sqrt[2]{25}\cdot\sqrt[2]{x}}=\frac{3\cdot 2\cdot\sqrt[3]{x}\cdot\sqrt[4]{x}}{5\cdot\sqrt[2]{x}}$. This expression may be written as $\frac{6x^{\frac{1}{3}+\frac{1}{4}-\frac{1}{2}}}{5}$ and simplified to $\frac{6x^{\frac{1}{12}}}{5}$. **Incorrect Response: A.** This response may reflect the misconception that $\frac{\sqrt[3]{x}\sqrt[4]{x}}{\sqrt{x}}=\sqrt[\left(\frac{3\cdot 4}{2}\right)]{x}$. **Incorrect Response: B.** This response may be the result of combining the reasoning described in response A along with the errors $\sqrt[3]{27}$ and $\sqrt[4]{16}=4$. **Incorrect Response: D.** This response may reflect the incorrect evaluation of the radicals described in response B.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
7		D	Objective 002

Correct Response: D. The vector representing z = –1 + i is in the second quadrant with horizontal component –1 and vertical component 1. Its magnitude (its distance from the origin) is $\sqrt{2}$, and its polar angle is 135° or $\frac{3\pi}{4}$ radians. Thus, the polar form of z is $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$. By de Moivre's theorem, $z^6 = (\sqrt{2})^6\left(\cos\frac{18\pi}{4} + i\sin\frac{18\pi}{4}\right)$. This is equivalent to $z^6 = 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ because the angles of $\frac{18\pi}{4}$ and $\frac{\pi}{2}$ are coterminal (i.e., they originate from the same position, the positive x-axis, and end at the same location on the unit circle). Since $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$, $z^6 = 8(0 + i) = 8i$.

Incorrect Response: A. This response may be the result of using a polar angle of $\frac{\pi}{4}$ rather than $\frac{3\pi}{4}$ and confusing the cosine and sine values. This angle leads to $z^6 = 8\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$, and the incorrect evaluation of the trigonometric functions as $\cos\frac{3\pi}{2} = -1$ and $\sin\frac{3\pi}{2} = 0$ yields –8.

Incorrect Response: B. This response may be the result of using a polar angle of $\frac{\pi}{4}$ rather than $\frac{3\pi}{4}$, so $z^6 = 8\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$. Using $\cos\frac{3\pi}{2} = 0$ and $\sin\frac{3\pi}{2} = -1$ yields $-8i$.

Incorrect Response: C. This response may be the result of using the correct polar angle but then confusing the values of cosine and sine in the last step.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
			That is, evaluate $z^6 = 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ using $\cos\frac{\pi}{2} =$ 1 and $\sin\frac{\pi}{2} = 0$ to obtain $z^6 = 8$.
8		B	Objective 002 **Correct Response: B.** The first four terms are shown below for $c = i$. 1st term: i 2nd term: $i^2 + i = -1 + i$ 3rd term: $(-1 + i)^2 + i = 1 - 2i + i^2 + i = -i$ 4th term: $-(i)^2 + i = -1 + i$ The second and fourth terms have a nonzero real part. **Incorrect Response: A.** This response may be the result of a misconception with complex numbers or an error applying the algorithm shown. **Incorrect Response: C.** This response may be the result of a misconception with complex numbers or an error applying the algorithm shown. **Incorrect Response: D.** This response may be the result of a misconception with complex numbers or an error applying the algorithm shown.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
9		B	Objective 002

Correct Response: B. Let the column vector $\begin{bmatrix} x \\ y \end{bmatrix}$ represent a vertex on the square shown. The result of the matrix multiplication

$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \bullet x + 0 \bullet y \\ 0 \bullet x + -1 \bullet y \end{bmatrix} = \begin{bmatrix} 2x \\ -y \end{bmatrix}$$

can be summarized in arrow notation as $(x, y) \rightarrow (2x, -y)$. This multiplication moves vertices A (1, 0), B (2, 0), C (2, 1), and D (1, 1) to A' (2, 0), B' (4, 0), C' (4, –1), and D' (2, –1).

Incorrect Response: A. This response does not show the value of the x-coordinates being doubled. Instead, the width of the square is doubled while the lower left vertex is left anchored at its original location.

Incorrect Response: C. The transformation in this response represents a reflection over the y-axis and a vertical stretch that doubles the value of each y-coordinate. This could be described by the matrix multiplication

$$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ 2y \end{bmatrix}$$

Incorrect Response: D. The transformation in this response shows a horizontal shift of 2 units left and a vertical stretch that doubles the value of each y-coordinate. This is not consistent with the transformation matrix provided.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
10		B	Objective 002 **Correct Response: B.** The vector (3, 4) extends from the point (0, 0) to the point (3, 4) and has a magnitude of $\sqrt{3^2 + 4^2}$ = 5 miles. The boat traveled 50 miles, which is 10 times the magnitude of this vector, in this direction. Therefore, the boat is located at point (30, 40). From here the boat travels directly to point (50, 48). The vector (50 – 30, 48 – 40) = (20, 8) describes the direction it will travel. This may be scaled down by a factor of 4 to obtain (5, 2). **Incorrect Response: A.** Consider a proof by contradiction: If this were true, then (30, 40) + (*a*, *a*) = (50, 48) for some value of *a*. No such value exists. Therefore, the boat will not reach the island by traveling along vector (1, 1). **Incorrect Response: C.** This response may result from the reasoning described for response D along with a transposition of the vector components. **Incorrect Response: D.** This response may result from reasoning that the boat has reached (6, 8) after 2 hours. The misconception is that the boat would only have traveled $\sqrt{6^2 + 8^2}$ = 10 miles if this were true. The vector (50 – 6, 48 – 8) = (44, 40) = (11, 10) describes the direction the boat would travel to reach the island.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
11		A	Objective 002

Correct Response: A. The matrix multiplication *AB* is performed as follows:

$$\begin{bmatrix} 10 & 3 & 10 \\ 0 & 10 & 10 \\ 2 & 0 & 5 \\ 6 & 5 & 10 \\ 4 & 8 & 12 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 10 \cdot 5 + 3 \cdot 10 + 10 \cdot 20 \\ 0 \cdot 5 + 10 \cdot 10 + 10 \cdot 20 \\ 2 \cdot 5 + 0 \cdot 10 + 5 \cdot 20 \\ 6 \cdot 5 + 5 \cdot 10 + 10 \cdot 20 \\ 4 \cdot 5 + 8 \cdot 10 + 12 \cdot 20 \end{bmatrix} = \begin{bmatrix} 280 \\ 300 \\ 110 \\ 280 \\ 340 \end{bmatrix}$$

The elements of this column matrix represent the total cost of all meals for each of the five days.

Incorrect Response: B. This response would follow if matrix *B* were written as a different matrix:

$$\begin{bmatrix} 10 & 3 & 10 \\ 0 & 10 & 10 \\ 2 & 0 & 5 \\ 6 & 5 & 10 \\ 4 & 8 & 12 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix} = \begin{bmatrix} 10 \cdot 5 + 0 + 0 & 0 + 3 \cdot 10 + 0 & 0 + 0 + 10 \cdot 20 \\ 0 \cdot 5 + 0 + 0 & 0 + 10 \cdot 10 + 0 & 0 + 0 + 10 \cdot 20 \\ 2 \cdot 5 + 0 + 0 & 0 + 0 \cdot 10 + 0 & 0 + 0 + 5 \cdot 20 \\ 6 \cdot 5 + 0 + 0 & 0 + 5 \cdot 10 + 0 & 0 + 0 + 10 \cdot 20 \\ 4 \cdot 5 + 0 + 0 & 0 + 8 \cdot 10 + 0 & 0 + 0 + 12 \cdot 20 \end{bmatrix} = \begin{bmatrix} 50 & 30 & 200 \\ 0 & 100 & 200 \\ 10 & 0 & 100 \\ 30 & 50 & 200 \\ 20 & 80 & 240 \end{bmatrix}$$

Incorrect Response: C. This response would follow if the result from incorrect response B were left multiplied by a row of 1s:

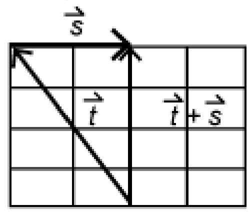
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 50 & 30 & 200 \\ 0 & 100 & 200 \\ 10 & 0 & 100 \\ 30 & 50 & 200 \\ 20 & 80 & 240 \end{bmatrix} = \begin{bmatrix} 110 & 260 & 940 \end{bmatrix}$$

To see why this is true, consider the calculation that resulted in 110: $50 \cdot 1 + 0 \cdot 1 + 10 \cdot 1 + 30 \cdot 1 + 20 \cdot 1$.

Incorrect Response: D. This response could be obtained by totaling the elements in the column vector shown in the correct response. This would occur if the column vector in that result were left-multiplied by a row matrix of 1s:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 280 \\ 300 \\ 110 \\ 280 \\ 340 \end{bmatrix} = [280 + 300 + 110 + 280 + 340]$$

$$= [1310]$$

Question Number	Your Response	Correct Response	Related Objectives and Rationale
12		A	Objective 002 **Correct Response: A.** The vectors may be added together by using the head-to-tail method, as shown. The resultant vector is vertical because the horizontal components of **t** and **s** are equal and opposite.  **Incorrect Response: B.** This vector represents –(**t** + **s**). **Incorrect Response: C.** This vector represents (**t** + 2**s**). **Incorrect Response: D.** This vector represents –**t**.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
13		B	Objective 003

Correct Response: B. One strategy is to multiply by one in the form of $\frac{x^2}{x^2}$, then $\frac{x^{-1} - x}{x^{-2} - x^2} \cdot \frac{x^2}{x^2} = \frac{x - x^3}{1 - x^4}$. Factor the numerator and denominator to get $\frac{x(1 - x^2)}{(1 + x^2)(1 - x^2)}$, which simplifies to yield $\frac{x}{1 + x^2}$.

Incorrect Response: A. This response may be the result of clearing fractions in both numerator and denominator by erroneously using different multipliers. Consider the expression written as

$$\frac{x^{-1} - x}{x^{-2} - x^2} = \frac{\frac{1}{x} - x}{\frac{1}{x^2} - x^2}.$$

If the numerator is multiplied by x and the denominator is multiplied by x^2, the expression becomes $\frac{1 - x^2}{1 - x^4}$. When the denominator is factored to get $(1 + x^2)(1 - x^2)$, the expression simplifies to $\frac{1}{1 + x^2}$.

Incorrect Response: C. This response may be the result of erroneously rewriting the given expression as the difference $\frac{x^{-1}}{x^{-2}} - \frac{x}{x^2}$ which then simplifies to $x - \frac{1}{x}$ and can be rewritten as $\frac{x^2 - 1}{x}$.

Incorrect Response: D. This response may be the result of erroneously rewriting the given expression as $\frac{x^{-1}}{x^{-2}} + \frac{x}{x^2}$, which simplifies to $x + \frac{1}{x}$ and can be rewritten as $\frac{x^2 + 1}{x}$.

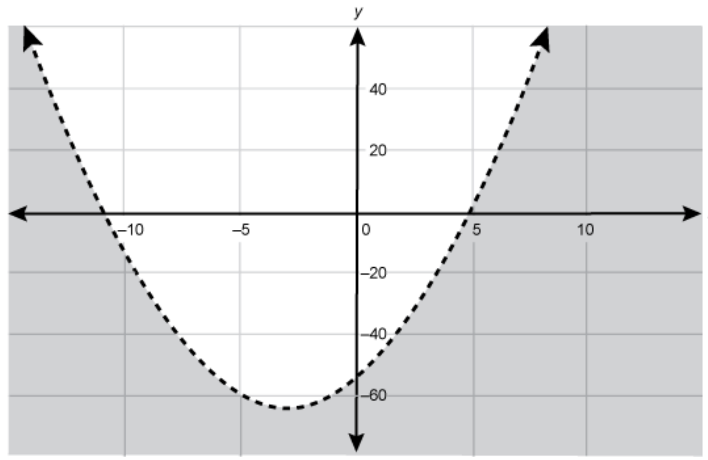
Question Number	Your Response	Correct Response	Related Objectives and Rationale
14		A	Objective 003 **Correct Response: A.** The inequality 6*Y* + 9*A* ≥ 1800 represents the student's goal for ticket sales. Solving the inequality for *A* yields the correct result: $9A \geq 1800 - 6Y \rightarrow A \geq \frac{1800}{9} - \frac{6}{9}Y \rightarrow A \geq 200 - \frac{2}{3}Y \rightarrow A \geq -\frac{2}{3}Y + 200$ **Incorrect Response: B.** This response may be the result of transposing the variables (charging $6 for an adult ticket and $9 for a youth ticket) and then solving the equation 9*Y* + 6*A* ≥ 1800 for *A*. **Incorrect Response: C.** This response may be the result of manipulating the inequality incorrectly. Subtracting 6Y from both sides yields 9*A* ≥ –6*Y* + 1800. Response C may be the result of dividing only the value of 1800 by 9 instead of dividing the entire equation by 9. **Incorrect Response: D.** This response may be the result of transposing the variables in the original equation and then incorrectly solving for *A* by only dividing 1800 by 6.
15		B	Objective 003 **Correct Response: B.** The associative property relates to the grouping of addends or factors. For multiplication: if *a*, *b*, and *c* are real numbers, then *a*(*bc*) = (*ab*)*c*. **Incorrect Response: A.** The commutative property relates to the order in which numbers are added or multiplied. For multiplication: if *a* and *b* are real numbers, then *a* • *b* = *b* • *a*. **Incorrect Response: C.** The distributive property describes how multiplication can be applied to the sum or difference of two (or more) terms: if *a*, *b*, and *c* are real numbers, then *a*(*b* ± *c*) = *ab* ± *ac*. **Incorrect Response: D.** The transitive property of equality states: if *a*, *b*, and *c* are real numbers and if *a* = *b* and *b* = *c*, then *a* = *c*.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
16		A	Objective 003 **Correct Response: A.** The technique of completing the square can be used to derive the quadratic formula $x = \frac{b^2 \pm \sqrt{b^2 - 4ac}}{2a}$. To obtain this formula from $ax^2 + bx + c = 0$, subtract c from each side of the equation and then divide each term by a: $x^2 + \frac{b}{a}x = \frac{-c}{a}$. The left side of this equation can be factored as $(x + k)^2$ if it is written as a trinomial of the pattern $x^2 + 2kx + k^2$. If $\frac{b}{a} = 2k$, then $k = \frac{b}{2a}$ and $k^2 = \frac{b^2}{4a^2}$. It follows that the equation may be written as $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$, which is the correct response. For additional context, factor the left side, take the square root, and solve for x: $(x + \frac{b}{2a})^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \rightarrow x + \frac{b}{2a} = \pm\sqrt{(\frac{b^2}{4a^2} - \frac{c}{a})} \rightarrow x = -\frac{b}{2a} \pm \sqrt{(\frac{b^2 - 4ac}{4a^2})}$. This is equivalent to the form of the quadratic formula shown above. **Incorrect Response: B.** This response is the result of squaring $\frac{b}{a}$ instead of $\frac{b}{2a}$ and adding $\frac{b^2}{a^2}$ to both sides of $x^2 + \frac{b}{a}x = \frac{-c}{a}$. The left side of this equation does not factor in the form $(x + k)^2$. **Incorrect Response: C.** This response is the result of forgetting to divide $-c$ by a, but squaring $\frac{b}{2a}$ correctly and adding it to both sides of $x^2 + \frac{b}{a}x = -c$. **Incorrect Response: D.** This response is the result of making both errors in responses B and C.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
17		D	Objective 003 **Correct Response: D.** The complex roots of a polynomial occur in conjugate pairs: If $3 + i$ is a root, then $3 - i$ is also a root. The product $(x - (3 + i))(x - (3 - i)) = x^2 - 6x + 10$ must be a quadratic factor of $P(x)$. **Incorrect Response: A.** This response is the result of pairing the –2 root with a +2 root and finding the product $(x + 2)(x - 2)$. The result might be a factor of $P(x)$, but it does not have to be. **Incorrect Response: B.** This response is the result of calculating $(x + 2)^2$, which might be a quadratic factor of $P(x)$, but it does not have to be. **Incorrect Response: C.** This response is the result of identifying the conjugate of $3 + i$ as $3 - i$, finding the product $(x - (3 + i))(x - (-3 - i))$, and rewriting the $-6i$ term as $-6x$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
18		A	Objective 003

Correct Response: A. The expression $x^2 + 6x - 55$ factors into the product $(x - 5)(x + 11)$. If $x < -11$, then $(x + 11)$ and $(x - 5)$ will both be negative numbers, which results in a positive product. If $x > 5$, then both binomials are positive numbers, so the result is a positive product. This may also be seen graphically:



Incorrect Response: B. This inequality is true only if $-11 < x < 5$ since this leads to a negative product of the binomials $(x - 5)$ and $(x + 11)$.

Incorrect Response: C. The expression $x^2 - 6x - 55$ factors into the product $(x - 11)$ and $(x + 5)$. For $x < -11$, both binomials are negative numbers, which produces a positive result. However, when $x > 5$, there are values for which $(x - 11)$ is negative and $(x + 5)$ is positive. The inequality $x^2 - 6x - 55 > 0$ does not hold true for those values of x.

Incorrect Response: D. The expression $x^2 - 6x - 55$ factors into the product $(x - 11)$ and $(x + 5)$. For $x < -11$, both binomials must be negative numbers, which implies a positive product.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
19		C	Objective 004 **Correct Response: C.** In Stage 1, $\frac{1}{9}$ of the total square is shaded. In Stage 2, each unshaded ninth of the square is divided into 9 again, so each of the new smaller shaded squares represents $\left(\frac{1}{9}\right)^2$ of the original area. Since 8 of those squares are shaded, the sum of the smaller squares in Stage 2 is $8 \cdot \left(\frac{1}{9}\right)^2$, and the total shaded area is the sum of Stage 1 and Stage 2: $\frac{1}{9} + 8 \left(\frac{1}{9}\right)^2$. Each subsequent iteration will introduce 8 times as many shaded squares as the previous iteration, with each new square having $\frac{1}{9}$ the previous size. This results in the expression $\frac{1}{9} + 8 \cdot \left(\frac{1}{9}\right)^2 + 8^2 \cdot \left(\frac{1}{9}\right)^3 + 8^3 \cdot \left(\frac{1}{9}\right)^4 + \ldots$ **Incorrect Response: A.** This expression implies that only one smaller square from a divided ninth is added in each stage after Stage 1. **Incorrect Response: B.** This expression does not describe the pattern because the size of the shaded sections in Stage 1, $\frac{1}{9}$, does not decrease as the pattern continues. Note that the sum of this expression exceeds 1 by Stage 3. **Incorrect Response: D.** After Stage 1, the pattern shades 8 smaller squares around each square drawn in the last round, so this number grows exponentially as 8^{n-1}. This expression in this response describes the pattern as adding 9 squares for each stage after Stage 1.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
20		C	Objective 004 **Correct Response: C.** If the pattern continues, then the function repeats itself every 2 units (i.e., the graph is periodic with a period of 2). This means that any multiple of 2 may be added or subtracted from *x* without changing the value of the function $f(x) = f(x + 2n)$, where n is an integer. Subtracting 16 (a multiple of 2) from $\frac{35}{2}$ yields $\frac{3}{2}$. Therefore, $f\left(\frac{35}{2}\right) = f\left(\frac{3}{2}\right)$. **Incorrect Response: A.** This response may be the result of misinterpreting the period of the function to be equal to 3: $\frac{35}{2} - 3(5) = 2.5 = \frac{5}{2}$. **Incorrect Response: B.** This response may be the result of misinterpreting the period of the function to be equal to 1.5: $\frac{35}{2} - 1.5(11) = 1$. **Incorrect Response: D.** This response may be the result of mistaking the period of the function to be the equivalence value.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
21		C	Objective 004 **Correct Response: C.** A function is a relation that assigns each element of set X (called the "domain," which is commonly referred to as "input values" or "*x*-coordinates") to exactly one element of set Y (called the "range," which is commonly referred to as "output values" or "*y*-coordinates"). For this graph, each element in the domain {–3, –2, 0, 2, 3, 5} is assigned exactly one value in the range. **Incorrect Response: A.** The elements –3 and 2 in the domain are both assigned to more than one value in the range. Functions do not have more than one *y*-value assigned to each *x*-value. **Incorrect Response: B.** The element 4 in the domain is assigned to more than one value in the range. Functions do not have more than one *y*-value assigned to each *x*-value. **Incorrect Response: D.** Solving the equation for y produces $y = \pm\sqrt{x + 1}$, so y generally has 2 values assigned to each value of x. For example, when $x = 3$, y could have the value of 2 or –2 . Functions do not have more than one *y*-value assigned to each *x*-value.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
22		B	Objective 004 **Correct Response: B.** The function $f(x) = x^2 + 2x + 5$ is a parabola that opens upward, with a vertex located at $(x, f(x))$ where $x = \left(\frac{-b}{2a}\right) = \frac{-2}{2(1)} = -1$. The parabola has a vertex at $(-1, f(-1))$ or $(-1, 4)$. The range of the parabola is $[4, \infty)$ (i.e., $f(x) \geq 4$). The composition $g(f(x))$ means that this range becomes the set of possible inputs for $g(x)$. The values that $g(x)$ may reach given these inputs extends from $\sqrt{4}$ to infinity. Therefore, the range of $h(x) \geq 2$. **Incorrect Response: A.** This response represents the domain of $g(x)$, not $h(x)$. **Incorrect Response: C.** This response could be the result of an error when solving the equation $x^2 + 2x + 5 \geq 0$ by completing the square. **Incorrect Response: D.** This response represents the domain of the function $f(x)$, not the range of $h(x)$.
23		C	Objective 004 **Correct Response: C.** Applying the inverse of a function, $f^{-1}(x)$, to the output of the original function, $f(x)$, yields the input for the original function, x. **Incorrect Response: A.** This response is the result of the misconception that $f^{-1}(x) = f(-x)$. Note that $f^{-1}(f(x)) = (2 - x) + 2$ does not return x. **Incorrect Response: B.** This response is the result of the misconception that the x variables and constants swap positions in an inverse function. Note that $f^{-1}(f(x)) = 2^{(x^2)}$ does not return x. **Incorrect Response: D.** This response reflects the misconception that applying the inverse of a function to the original function yields the multiplicative identity element.

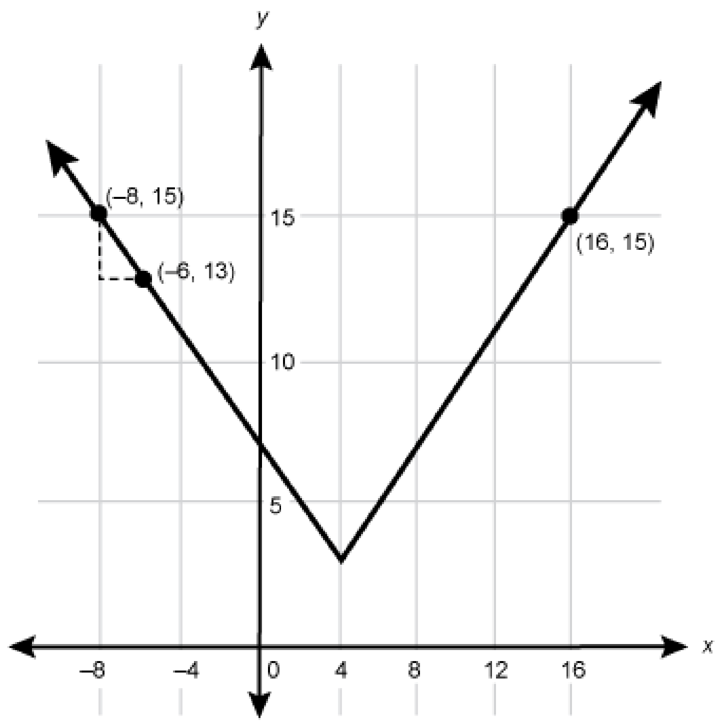
Question Number	Your Response	Correct Response	Related Objectives and Rationale
24		D	Objective 004 **Correct Response: D.** Arithmetic sequences have a common difference between any two consecutive terms. Since $a_8 = \frac{41}{2}$ and $a_{16} = \frac{81}{2}$, the common difference is equal to $\frac{1}{8}\left(\frac{81}{2} - \frac{41}{2}\right) = \frac{1}{8}\left(\frac{40}{2}\right) = \frac{1}{8}(20) = \frac{20}{8} = \frac{5}{2}$. Using the common difference to work backward from the 8th term: $a_1 = a_8 - 7\left(\frac{5}{2}\right) = \left(\frac{41}{2}\right) - 7\left(\frac{5}{2}\right) = \frac{6}{2} = 3$. **Incorrect Response: A.** This response is a misinterpretation of the characteristics of an arithmetic sequence, and it leads to the terms growing in such a way that there is not a common difference between consecutive terms. **Incorrect Response: B.** This response uses the correct common difference, but the incorrect first term is the result of subtracting $a_8 - 8\left(\frac{5}{2}\right)$. **Incorrect Response: C.** This response is a misinterpretation of the characteristics of an arithmetic sequence, and it leads to the terms growing in such a way that there is not a common difference between consecutive terms.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
25		B	Objective 005 **Correct Response: B.** If the data fit a linear function, then the points (20, 10,620) and (30, 14,940) are solutions to the linear equation $y = mx + b$, where m is the unit cost of a single air conditioner, x is the number of air conditioners, and b is a fixed cost. The difference \$14,940 – \$10,620 = \$4,320 represents the cost of 10 air conditioners (note that subtracting these values eliminated b). It follows that 1 air conditioner costs \$432, 6 cost \$2,592, and 36 cost \$14,940 + \$2,592. **Incorrect Response: A.** This represents \$432(36), which does not account for the fixed costs to manufacture the air conditioners. **Incorrect Response: C.** This response may be the result of incorrectly computing the cost to manufacture an air conditioner by dividing 10,620 by 20 to obtain a cost of \$531 per air conditioner, then computing 14,940 + 6(531) = 18,126. This computation disregards any fixed costs required to manufacture the air conditioners. **Incorrect Response: D.** This response may be the result of a combination of both a miscalculation for the cost to manufacture one air conditioner and a disregard for any initial fixed costs in the process.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
26		B	Objective 005 **Correct Response: B.** Set the two equations equal to each other to determine the values for *m* that do have points of intersection: $mx + 3 = -x^2 + 3x + 2 \rightarrow x^2 + (m - 3)x + 1 = 0$. Substitute the coefficients into the quadratic formula to obtain $x = \frac{-(m-3) \pm \sqrt{(m-3)^2 - 4(1)(1)}}{2(1)}$. Real values of *x* correspond to points of intersection; therefore, determine where the radicand is negative: $(m - 3)^2 - 4 < 0 \rightarrow m^2 - 6m + 5 < 0 \rightarrow (m - 1)(m - 5) < 0$. The product on the left side will only be negative when one factor is positive and the other negative. This occurs in the interval $1 < m < 5$. **Incorrect Response: A.** This inequality could be the result of solving $(m + 3)^2 - 4 < 0$. **Incorrect Response: C.** This inequality could be the result of solving $(m + 3)^2 - 4 > 0$. **Incorrect Response: D.** This inequality represents the values for *m* for which the two graphs will intersect.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
27		B	Objective 005

Correct Response: B. The absolute value function $y = |x + b| + c$ is symmetrical about $(–b, c)$, the point of its vertex. It is a piece-wise function made of two linear functions: the linear function extending toward positive x has a slope of 1, and the linear function extending toward $–x$ has a slope of –1. Use the latter slope to move from the point (–6, 13) to (–6 – 2, 13 + 2) = (–8, 15), as shown in the following graph.



This point has the same y-coordinate as (16, 15), so the line of symmetry for this graph is located at the average of these x-coordinates: $\frac{16 + (–8)}{2} = 4$. Therefore, the vertex is at $x = 4$. Its y-coordinate may be determined either by moving 10 units to the right from (–6, 13) along a slope of –1, (–6 + 10, 13 – 10), or by moving 12 units to the left from (16, 15) along a slope of –1, (16 – 12, 15 – 12). Both paths lead to (4, 3). Alternatively, for a more direct solution, describe each side of the absolute value equation by writing linear equations in point-slope form and find their point of intersection by solving the system. The equations $y – (15) = 1 \cdot (x – 16)$ and $y – (13) = –1 \cdot (x – (–6))$ have a common solution at (4, 3).

Incorrect Response: A. This point represents the y-intercept for the function.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
			Incorrect Response: C. This response may be the result of finding the midpoint between the two points. **Incorrect Response: D.** This response may be the result of multiple errors that build upon each other, such visualizing the absolute value function as opening downward along with arithmetic errors related to the slope. Notice that a line segment that connects points (–6, 13) and (8, 23) does not have a slope equal to 1 (or –1).
28		B	Objective 005 **Correct Response: B.** A parabola written in the standard form of $f(x) = ax^2 + bx + c$ has a vertex located at $x = -\frac{b}{2a}$. The vertex for $f(x)$ is located at $x = -\frac{-24}{4} = 6$. The positive leading coefficient of the function gives the graph upward concavity—this means that its vertex is located at the minimum value of the function and the function always increases in value for values of x that move away from it in either direction. For this reason, $x = -3$, being the value furthest away from the vertex in the interval $-3 \le x \le 5$, generates the highest value for $f(x)$: $f(-3) = 2(-3)^2 - 24(-3) + 6 = 96$. **Incorrect Response: A.** This response may be the result of misinterpreting the given interval in the problem: $176 = f(-5)$. **Incorrect Response: C.** This response may be the result of the misconception that the function has a maximum value on the interval when $x = 5$. This may be the result of interpreting the parabola as opening downward instead of opening upward. **Incorrect Response: D.** The vertex is located at $x = 6$, $f(6) = -66$. Note that this value is outside the interval of values to be considered.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
29		C	Objective 005 **Correct Response: C.** The inequality $2x + y \geq 0$ can be expressed as $y \geq -2x$. In a graph, the boundary of this inequality appears as a solid line passing through the origin with a slope of –2. The boundary of the other inequality, $x - y < 1$, is graphed with a dashed line and with intercepts at $x = 1$ and $y = -1$. This inequality can also be expressed as $y > x - 1$. Together, these two inequalities admit solutions in the coordinate plane that are above or along the solid line and above the dashed line. The shaded region represents all values that create true statements with both inequalities. **Incorrect Response: A.** This represents the system of inequalities $2x + y \geq 0$ and $x - y > 1$. **Incorrect Response: B.** This represents the system of inequalities $2x + y \leq 0$ and $x - y > 1$. **Incorrect Response: D.** This represents the system of inequalities $2x + y \leq 0$ and $x - y < 1$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
30		A	Objective 005 **Correct Response: A.** Given the scenario, only positive values for *p* and *b* make sense, so all graphs will be limited to quadrant 1. Constraints on the bakery's production capabilities limit the number of bagels and pretzels it can make each day. These constraints may be expressed as a system of linear inequalities. The inequality $3b + p \leq 12$ describes the constraint that the time to bake the bagels and pretzels cannot exceed 12 hours per day. Similarly, $2b + 4p \leq 16$ describes the constraint that the time to prepare the bagels and pretzels cannot exceed 16 hours per day. The correct lines and shading for these combined inequalities form the feasibility region shown in response A. The dashed lines represent the objective function to be maximized (i.e., profit, in this context). Combinations of bagels and pretzels that lie on the same dashed line yield equivalent profits for the bakery, and the further a dashed line is from the origin, the greater its profit. (These lines represent the equation $60b + 50p = k$ for different values of *k,* where *k* represents a given profit—lines further from the origin show greater values for *k*). The maximum achievable profit occurs at the vertex where the lines bounding the feasible region intersects the second dashed line. **Incorrect Response: B.** The feasible region in this response represents the system of inequalities $3b + p \leq 12$ **or** $2b + 4p \leq 16$. The feasible region in the correct response represents the system of inequalities $3b + p \leq 12$ **and** $2b + 4p \leq 16$. The bakery cannot produce the combinations of pretzels and bagels beyond the feasible region shown in the correct response. **Incorrect Response: C.** This response assigns the coefficients for the bagel variables to the pretzel variables and vice versa: $3b + p \leq 12$ and $4b + 2p \leq 16$. **Incorrect Response: D.** This response combines the errors described in responses B and C.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
31		A	Objective 006 **Correct Response: A.** If the piece of equipment decreases in value by 6% each year, then the equipment retains 100% – 6% = 94% of its value each year. 42,000(0.94) can be used to find the equipment's value at the end of the first year. For each subsequent year, the value is 94% of the value from the previous year. This constant ratio indicates that the value of the equipment decays exponentially. The value of the equipment can be determined using the expression $42{,}000(0.94)^n$. **Incorrect Response: B.** This equation is incorrect because it assumes a constant (rather than exponential) rate of change in the value of the equipment over time. Additionally, using this formula generates a value greater than the initial value for year 2, as $42{,}000(0.94 \times 2) = 42{,}000 \times 1.88 = 78{,}960$. **Incorrect Response: C.** This response is based on a misinterpretation of the correct equation $V_n = 42{,}000(1 - 0.06)^n$. **Incorrect Response: D.** This equation shows a linear decrease of $2,520 (6% of the original value) each year instead of the exponential decay of 6% of the value each year.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
32		B	Objective 006 **Correct Response: B.** The general equation of an exponential function with the *x*-axis as its horizontal asymptote is $y = ab^x$, where b is a real number ≠ 1. Substituting (0, 3) into the equation gives $3 = ab^0$; $b^0 = 1$, so $a = 3$. Substituting $(-2, \frac{27}{4})$ generates the equation $\frac{27}{4} = 3b^{-2}$, which is equivalent to $\frac{9}{4} = \frac{1}{b^2}$. It follows that $b = \frac{2}{3}$. **Incorrect Response: A.** This response could be the result of noticing that the *y*-intercept is given and incorrectly writing the exponential function $y = b^x + 3$. Solving $\frac{27}{4} = b^{-2} + 3$, results in $b = \frac{2}{\sqrt{15}}$. **Incorrect Response: C.** This response could be the result of thinking as in Response A above, but ignoring or making an error regarding the negative exponent. If $\frac{27}{4} = b^2 + 3$, then $b = \frac{\sqrt{15}}{2}$. **Incorrect Response: D.** This response could be the result of using $y = ab^x$, correctly identifying $a = 3$, then ignoring or making an error regarding the negative exponent. If $\frac{27}{4} = 3b^2$, then $b = \frac{3}{2}$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
33		A	Objective 006

Correct Response: A. One way to determine the correct response is to take the graph of $y = 2^x$, reflect it over the y-axis to obtain $y = 2^{-x}$, and then shift that graph one unit to the left to get the graph of $f(x) = 2^{1-x}$. The inverse will be represented by the graph resulting from reflecting the graph of $f(x) = 2^{1-x}$ over the line $y = x$. This is graph A.
Alternatively, make a table of values for $f(x) = 2^{1-x}$.

x	–2	–1	0	1	2
$f(x)$	8	4	2	1	0.5

The inverse function maps the $f(x)$ values back to the x values. The table for $f^{-1}(x)$ follows.

x	8	4	2	1	0.5
$f^{-1}(x)$	–2	–1	0	1	2

These are coordinates of points on graph A.

Incorrect Response: B. This graph is the reflection of the graph of $f(x) = 2^{1-x}$ over the y-axis.

Incorrect Response: C. This graph is the reflection of the graph of $f(x) = 2^{1-x}$ over the line $y = -x$.

Incorrect Response: D. This graph is the reflection of the graph of $f(x) = 2^{1-x}$ over the x-axis.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
34		A	Objective 006 **Correct Response: A.** The domain of $f(x) = \log_4(x + 2)$ is the set of real numbers for which $x + 2 > 0$. Thus $x > -2$. Testing values of x close to –2 indicates that $x = -2$ is a vertical asymptote. For $x = 0$, $\log_4(2) = \frac{1}{2}$ because $4^{\frac{1}{2}} = 2$. Then $\left(0, \frac{1}{2}\right)$ is the y-intercept. For $y = 0$, $\log_4(x + 2) = 0$ is equivalent to $x + 2 = 4^0$; $4^0 = 1$, so $x = -1$. Thus, the x-intercept is (–1, 0). **Incorrect Response: B.** This response may be a result of thinking that when $x = 0$, $\log_4(2) = -2$, possibly thinking that $4^{-2} = 2$. **Incorrect Response: C.** This response may be a result of thinking that when $x = -2$, $\log_4(0) = 0$ and since (–2, 0) is an intercept, then $x = -2$ cannot be a vertical asymptote, so the asymptote must then be $x = 2$. **Incorrect Response: D.** This response may be a result of combining the errors made in each of the previous responses.
35		C	Objective 006 **Correct Response: C.** If $\frac{\log_2(x-1)}{5} = 1$, then $\log_2(x - 1) = 5$. An equation of the form $\log_b N = p$, where b is the base and p is the exponent, may be equivalently written as the equation $b^p = N$. This statement says the exponent used on base 2 is 5, so $2^5 = x - 1$. **Incorrect Response: A.** This response confuses the base with the exponent. **Incorrect Response: B.** This response has the correct base, but confuses the result with the exponent. **Incorrect Response: D.** This response confuses the relationship of all three of the values.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
36		B	Objective 006 **Correct Response: B.** Substitute: $f\,(g(x)) = 3 \bullet 2^{\log_2\left(\frac{1}{3}x\right)} = 3 \bullet \frac{1}{3}x = x$. **Incorrect Response: A.** This response may reflect algebraic errors with a misconception that these operations cancel each other in a way that 1 is the result, such as with the erroneous claim that $\log_2 2^x = 1$. **Incorrect Response: C.** This response may be the result of performing several errors, such as $3 \bullet 2^{\log 2\left(\frac{1}{3}x\right)} = 3 \bullet 2^{\left(\frac{1}{3}x\right)} = 6^{\left(\frac{x}{3}\right)}$. **Incorrect Response: D.** This response may be the result of misunderstanding the relationship between exponents and logarithms: $3 \bullet 2^{\log_2\left(\frac{1}{3}x\right)} = 3 \bullet 2^{\left(\frac{x}{3}\right)}$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
37		C	Objective 006 **Correct Response: C.** Using the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the total amount, P is the principal, r is the interest rate as a decimal value, n is the number of times the account is compounded annually, and t is the number of years, $10{,}000\left(1 + \frac{0.05}{4}\right)^{4t} = 20{,}000 \rightarrow (1.0125)^{4t} = 2$. **Incorrect Response: A.** This response may be the result of applying the formula for continuously compounded interest, $P(t) = P_0e^{rt}$, and dividing the interest rate by 4. **Incorrect Response: B.** This response may be the result of applying the formula for continuously compounded interest, $P(t) = P_0e^{rt}$. **Incorrect Response: D.** This response may be the result of forgetting to divide the interest rate, r, by 4.
38		B	Objective 007 **Correct Response: B.** A rational function is a function of the form $R(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions, and q is not the zero polynomial. Graphs of rational functions may have vertical and horizontal asymptotes, and the graph in this response has one of each. **Incorrect Response: A.** This graph does not exhibit behavior consistent with a function that has a polynomial denominator. **Incorrect Response: C.** This piecewise-defined function holds a constant value in each stretch of its interval. This is not consistent with having a polynomial denominator. **Incorrect Response: D.** This is a graph of an absolute value function.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
39		D	Objective 007 **Correct Response: D.** Critical points include points at which the rate of change changes sign (also known as relative minimums or relative maximums). The function $f(x) = x(x^2 - a^2) = x(x - a)(x + a)$ has 3 zeroes: $x = 0$, $x = \pm a$. Based on the coefficient of x^3, the function is negative when $x < -a$, reaches 0 at $x = -a$, and then becomes positive. One critical point must exist between $x = -a$ and $x = 0$ because the value of the function becomes 0 again at the origin. A similar argument holds for positive values of x. The function dips, turns at a critical point, and then crosses into positive values beyond $x = a$. Alternatively, a critical point of a continuous function is a point at which the derivative is equal to zero or undefined. Given that $f(x) = x^3 - a^2x$, $f'(x) = 3x^2 - a^2$. Since a^2 is a positive constant, it follows that $f'(x)$ is equal to zero for two values of x: $x = \pm \frac{a}{\sqrt{3}}$. **Incorrect Response: A.** This may be the result of interpreting the function to be a parabola or an even polynomial function with symmetrical end behavior. **Incorrect Response: B.** $f(x) = x^3 - a^2x = x(x^2 - a^2)$. Using the Zero Product Property, we can state that the function has an x-intercept at $x = 0$ and, since a^2 is positive, $x^2 - a^2 = 0 \rightarrow x^2 = a^2 \rightarrow x = \pm a$ showing that this function has exactly 3 real roots. **Incorrect Response: C.** This may be the result of interpreting $f(x)$ to be a rational function. The function $f(x)$ is a cubic function that is defined for all values of x and y.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
40		D	Objective 007 **Correct Response: D.** The polynomial has roots at x = –4, –1, and 2. This indicates that it may be written as $P(x) = k(x + 4)^b(x + 1)^c(x - 2)^d$. The end behavior of any polynomial of degree n (which here is equal to $b + c + d$) should resemble the power function $f(x) = ax^n$. Because the function is shown increasing without bound when x approaches negative infinity and decreasing without bound as x approaches positive infinity, we can infer that the leading coefficient must be negative and that n is an odd power. **Incorrect Response: A.** $P(x) = -0.03(x - 4)(x - 1)(x + 2)$: This polynomial has roots at x = 4, 1, and –2, which is not consistent with the information shown. **Incorrect Response: B.** $P(x) = -0.03(x + 4)(x + 1)(x - 2)$: This polynomial has roots located at the correct positions, but the multiplicity of the roots at x = –4 and 2 are incorrect. The graph touches, but does not cross, the x-axis at these locations and this indicates these roots must have an even multiplicity (i.e., be raised to an even power). Each of the roots for this polynomial are of the first power, so the graph would cross the axis at these points. **Incorrect Response: C.** $P(x) = -0.03(x - 4)^2(x - 1)(x + 2)^2$: This polynomial has roots at x = 4, 1, and –2. This is not consistent with the information shown.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
41		D	Objective 007 **Correct Response: D.** This expression factors into $\frac{(x + 1)(x - 1)}{2(x + 1)(x - 2)}$. The numerator and denominator both contain the factor $(x + 1)$ and so the graph has a removable discontinuity at $x = -1$. The expression simplifies to $\frac{1}{2} \cdot \frac{x - 1}{x - 2}$ with the exclusion $x \neq 1$. Since $(x - 2)$ is a factor of the denominator, the graph has a vertical asymptote at $x = 2$. Also, because $\frac{x - 1}{x - 2}$ can never equal 1, the value of the function cannot be equal to $\frac{1}{2}$, which creates a horizontal asymptote in the graph along $y = \frac{1}{2}$. **Incorrect Response: A.** This graph has a defined value at $x = -1$ (i.e., 0). Its vertical asymptotes occur when the denominator equals zero (which does not include $x = 2$) and the horizontal asymptote is at $y = 0$. The horizontal asymptote may be determined by factoring the leading power of x from the numerator and denominator of the rational function: $\frac{x \cdot (1 + \frac{1}{x})}{x^2 \cdot (1 - \frac{4}{x} - \frac{4}{x^2})}$ behaves as $\frac{1}{x}$ for large values of x, and $\frac{1}{x}$ approaches 0 as x increases. **Incorrect Response: B.** This equation simplifies to $y = \frac{x + 1}{2(x + 1)(x + 2)} = \frac{1}{2(x + 2)}$. It has a point discontinuity at $x = -1$, a vertical asymptote at $x = -2$, and a horizontal asymptote at $y = 0$. **Incorrect Response: C.** This equation has a defined value at $x = -1$ (i.e., 0) and a vertical asymptote at $x = -2$. For very large positive or negative values of x, the equation is approximately equal to 1, which appears in its graph as a horizontal asymptote.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
42		C	Objective 007 **Correct Response: C.** The two vertices of the rectangle above the *x*-axis must be directly above those on the x-axis and share the same *y*-coordinate. The information given describes the first-quadrant vertex at (x, y), where it intersects the function $y = 36 - x^2$. Because this function is symmetric above the *y*-axis, the other upper-vertex coordinate must be located at ($-x$, y). The area of the rectangle is the product of its width, $2x$, and its height, $y = 36 - x^2$. This leads to the equation $2x(36 - x^2) = 160$, which simplifies as $x^3 - 36x + 80 = 0$. **Incorrect Response: A.** This equation can be obtained from $2(6)(36 - x^2) = 160$. In this response, the roots of the parabola are treated as if they represent the width of the rectangle, and the height is obtained from an unknown—and unrelated—value of *x*. **Incorrect Response: B.** This equation can be obtained from $2(6)(36 - x^2) = 80$. In this response, the roots of the parabola area treated as if they represent the width of the rectangle, and the height is obtained from an unknown—and unrelated—value of *x*. **Incorrect Response: D.** Describing the width of the rectangle as x rather than $2x$ leads to the equation $x(36 - x^2) = 160$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
43		B	Objective 007 **Correct Response: B.** Factor the polynomials in the equation to obtain $\frac{(x-5)(x+3)}{(x-3)(x+3)} = 0$. The factors of $x + 3$ cancel, and the equation $\frac{(x-5)}{(x-3)} = 0$ has the solution $x = 5$. **Incorrect Response: A.** These values for x represent exclusions from the domain of the function (because the rational expression is undefined there). A graph of the function $f(x) = \frac{x^2 - 2x - 15}{x^2 - 9}$ has a removable point discontinuity at $x = -3$ (where the factors of $x + 3$ cancel) and a vertical asymptote at $x = 3$. **Incorrect Response: C.** These values for x represent the solution to the equation, $x = 5$, and one of the values for which the rational expression is undefined. **Incorrect Response: D.** These values for x represent the solution to the equation, $x = 5$, and both values for which the rational expression is undefined.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
44		C	Objective 007 **Correct Response: C.** The second differences of a sequence defined by a quadratic function are constant and nonzero. The sequence of values produced by $f_3(x)$ has first differences of 3, 5, 7, 9... and this sequence has constant second differences of 2. **Incorrect Response: A.** The sequence of second differences for $f_1(x)$ are 3, 5, 7..., which is not constant. The third differences, however, are constant and this could indicate that $f_1(x)$ is a cubic polynomial. **Incorrect Response: B.** The sequence of first differences for $f_2(x)$ are constant and its second differences are 0. This indicates that $f_2(x)$ could be a linear function. **Incorrect Response: D.** The sequence of second differences for $f_4(x)$ are 1, 2, 3..., which is not constant. The third differences, however, are constant and this indicates that $f_4(x)$ may be a cubic polynomial.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
45		A	Objective 008 **Correct Response: A.** The angle $\frac{8\pi}{3}$ is coterminal with $\frac{2\pi}{3}$ because $\frac{8\pi}{3} - 2\pi = \frac{8\pi}{3} - \frac{6\pi}{3} = \frac{2\pi}{3}$ (i.e., the angles originate from the same position—the positive *x*-axis—and end at the same location on the unit circle). Use the relationships $\sec x = \frac{1}{\cos x}$ and $\cos(\pi - x) = -\cos x$ to evaluate the expression as follows: $\frac{1}{\cos\frac{2\pi}{3}} = \frac{1}{-\cos(\pi - \frac{2\pi}{3})} = -\frac{1}{\cos(\frac{\pi}{3})} = -\frac{1}{\frac{1}{2}} = -2$. **Incorrect Response: B.** This response could be the result of computing $-\cos\frac{\pi}{3}$ rather than $-\frac{1}{\cos\frac{\pi}{3}}$. **Incorrect Response: C.** This response could be the result of computing $\sin\frac{8\pi}{3}$. **Incorrect Response: D.** This response could be the result of computing $\csc\frac{8\pi}{3}$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
46		A	Objective 008 **Correct Response: A.** The minimum point of $y = \sin x$ over the interval $0 \le x \le 2\pi$ occurs at $x = \frac{3\pi}{2}$, where $\sin\frac{3\pi}{2} = -1$. For the minimum point to be located at $\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$ without a vertical shift or change in period, the function must horizontally shift (via a phase shift) to the right by $\frac{11\pi}{6} - \frac{3\pi}{2} = \frac{2\pi}{6} = \frac{\pi}{3}$. An amplitude of $\frac{1}{2}$ reduces the minimum value to $-\frac{1}{2}$. **Incorrect Response: B.** This response uses the correct amplitude, but the phase difference may be based on the minimum value of $\sin x$ occurring at $x = \pi$. **Incorrect Response: C.** This response uses the correct phase shift, but the phase difference may be based on an incorrect amplitude for $\sin x$. **Incorrect Response: D.** This response may combine the errors in responses B and C.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
47		B	Objective 008 **Correct Response: B.** The given information states that the average daily temperature for a city over the span of one year can be modeled by a cosine function with an average daily temperature of *T*. The additional parameters for the cosine function can be determined from the given information. A cosine function with a minimum value of 16° and a maximum value of 80° has an average value of $\frac{16° + 80°}{2} = 48°$ and an amplitude of $80° – 48° = 32°$. The natural period of a cosine function is 2π. Multiplying t by 2π compresses the period to 1 (i.e., it would have a period of 1 day in this context) and dividing $2\pi t$ by 365 stretches the period to 365 days. The cosine function must be shifted to the right by 212 days to make its maximum value occur on July 31, which is done by replacing t with $t – 212$. These combined details are captured in the function $T = 32 \cos\left[\frac{2\pi}{365}(t - 212)\right] + 48$. **Incorrect Response: A.** This response uses a cosine function with an average value of 32° and an amplitude of 48°, which is inconsistent with the information given. **Incorrect Response: C.** This response uses the reciprocal of the correct period and transposes the average daily temperature value with the amplitude, as described in response A. **Incorrect Response: D.** This response uses the reciprocal of the correct period.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
48		B	Objective 008 **Correct Response: B.** The equation $\sin(2x) = \pm\cos(2x)$ develops from taking the square root of both sides of the equation. Consider the positive root first. Dividing both terms in the original equation by $\cos(2x)$ yields $\tan(2x) = 1$. However, the full period of the tangent function is π, and so the full solution is $\tan(2x + n \cdot \pi) = 1$, where n is an integer. Apply the inverse tangent to both sides of the equation and solve for x: $2x + n \cdot \pi = \tan^{-1}(1) \rightarrow x = \frac{\pi}{8} + n \cdot \frac{\pi}{2}$. Apply similar reasoning to find the solutions to the negative root: $2x + n \cdot \pi = \tan^{-1}(-1) \rightarrow x = -\frac{\pi}{8} + n \cdot \frac{\pi}{2}$. The first positive solution to $\sin^2(2x) = \cos^2(2x)$ comes from the positive root solution $x = \frac{\pi}{8}$. The next positive solution to $\sin^2(2x) = \cos^2(2x)$ comes from the negative root solution $x = -\frac{\pi}{8} + 1 \cdot \frac{\pi}{2} = \frac{3\pi}{8}$. The difference between these roots is $\frac{\pi}{4}$. **Incorrect Response: A.** This may be the result of an error in deriving the correct equation, analyzing the relationship between multiple solutions, or applying the correct periodicity to the inverse functions. **Incorrect Response: C.** This may be the result of an error in deriving the correct equation, analyzing the relationship between multiple solutions, or applying the correct periodicity to the inverse functions. **Incorrect Response: D.** This response corresponds to the difference between two consecutive solutions for either one of the roots described in the correct response.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
49		C	Objective 008

Correct Response: C.

$$\frac{\tan^2 x}{1+\tan^2 x} = \frac{\tan^2 x}{\sec^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \sin^2 x$$

Incorrect Response: A. This may be the result of confusing tan x with cot x and evaluated $\frac{\cot^2 x}{1 + \cot^2 x}$.

Incorrect Response: B. This may be the result of substituting the denominator of the expression with $1 + \tan^2 x = \sec^2 x$ and the numerator with $\tan^2 x = \sec^2 x - 1$ to obtain $\frac{\sec^2 x - 1}{\sec^2 x}$. However, this expression does not simplify to $\sec^2 x$.

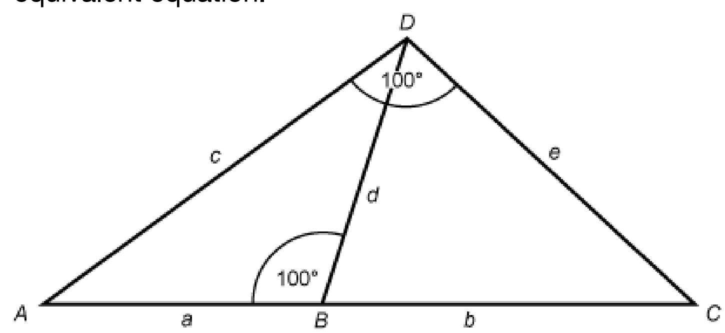
Incorrect Response: D. This may be the result of substituting the denominator of the expression with $1 + \cot^2 x = \csc^2 x$ and the numerator with $\cot^2 x = \csc^2 x - 1$ to obtain $\frac{\csc^2 x - 1}{\csc^2 x}$. However, this expression does not simplify to $\csc^2 x$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
50		B	Objective 008 **Correct Response: B.** This response follows from the transformation multiplying sin *x* by the identity property and expressing the cot *x* in terms of other functions: $\sin x \bullet \frac{\sin x}{\sin x} + \cos x \bullet \frac{\cos x}{\sin x} = \csc x$. **Incorrect Response: A.** This response is the result of multiplying sin *x* to only the left side of the equation. **Incorrect Response: C.** This response may represent the division of each term by cos *x* with an error on the right side of the equation: $\frac{\csc x}{\cos x} = \csc x \bullet \sec x \neq \frac{1}{\sin x}$. **Incorrect Response: D.** This response simplifies cos *x* • cot *x* in an incorrect manner.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
51		C	Objective 009 **Correct Response: C.** The octagon can be formed by cutting an isosceles right triangle from each corner of a square. The side of the octagon measures 1 foot and represents the hypotenuse of the triangle. The legs of the triangle have length $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$. The side of the square is then $1 + \frac{2\sqrt{2}}{2}$ or $1 + \sqrt{2}$. The area of the octagon is the area of the square minus the areas of the 4 triangles: $(1+\sqrt{2})^2 - 4\left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}\right) = 2\sqrt{2} + 2$. **Incorrect Response: A.** This response could be the result of subtracting the areas of the 4 triangles from the area of the square, but calculating the total area of the triangles as 2 rather than 1, thus subtracting 1 unit more than is required. **Incorrect Response: B.** This response could be the result of using the standard formula Area = $\frac{1}{2}$(apothem)(perimeter), but mistaking the length of the apothem as $\frac{1}{2} + \sqrt{2}$ rather than $\frac{1}{2} + \frac{\sqrt{2}}{2}$. **Incorrect Response: D.** This response could be the result of letting x represent the radius of the octagon and using the central angle to write $A = \frac{1}{2}x^2 \sin 45°$. This method uses the law of cosines to find x: $1 = 2x^2 - 2x^2\cos45°$ so $x^2 = \frac{1}{2-\sqrt{2}}$. When these were combined, a factor of $\frac{1}{2}$ was lost.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
52		C	Objective 009

Correct Response: C. Triangles *BDA* and *DCA* as labeled in the diagram shown below have two pairs of congruent angles: $\angle DAB \cong \angle DAC$ and $\angle DBA \cong \angle ADC$. Therefore, the angles are similar by AA, and this justifies the proportion $\frac{AC}{DC} = \frac{AD}{BD}$. Using the lengths provided, $\frac{a+b}{e} = \frac{c}{d}$. The equation $ce = ad + bd$ represents an equivalent equation.



Incorrect Response: A. This equation may be obtained from an incorrect observation that $\frac{c}{a} = \frac{e}{b}$.

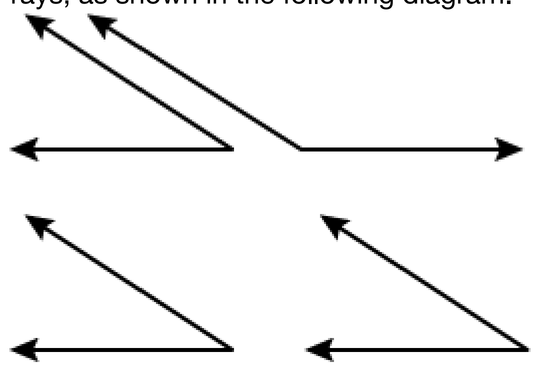
Incorrect Response: B. This simplifies to $a = b$ and could be obtained from a misconception that d bisects the bottom edge.

Incorrect Response: D. This equation is likely the result of incorrectly claiming pairs of edges represent corresponding parts of similar triangles.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
53		C	Objective 009 **Correct Response: C.** The volume of the original container can be found using the equation: $V_1 = \ell \times w \times d$. The volume of the new container can be found using the equation $V_2 = 4\ell \times 2w \times \frac{1}{2}d$. Regroup the factors and compare with V_1: $4 \times 2 \times \frac{1}{2} \times \ell \times w \times d = 4V_1$. The volume of the new container is 4 times the volume of the original container. **Incorrect Response: A.** This result could be due to a misconception where the effect of quadrupling the length is counteracted by doubling the width, so halving the depth produces a volume that is half of the original volume. **Incorrect Response: B.** This result could be due to a misconception where the net effect of quadrupling the length, doubling the width, and halving the depth leaves the volume unchanged. **Incorrect Response: D.** This result could be due to a computational error where the length is quadrupled, the width is also quadrupled, and the depth is halved.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
54		B	Objective 009 **Correct Response B:** The volume of the solids can be computed as shown. (Note that for the prism, properties of 30-60-90 triangles must be used to find the height of the triangle.) Square pyramid: $V = \frac{1}{3}x^2h$ Cone: $V = \frac{\pi}{3}x^2h$ Cylinder: $V = \pi r^2 \cdot h = \pi\left(\frac{x}{2}\right)^2 \cdot h = \frac{\pi}{4}x^2h$ Prism: $V = Bh = \left(\frac{x}{2} \cdot \frac{x\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} \cdot x^2h$ Since the factor x^2h is present in all cases, the volumes can be compared using the leading coefficients. The coefficient $\frac{\pi}{3}$ is the only coefficient greater than one, so the cone has the greatest volume. **Incorrect Response: A.** This response could be the result of misinterpreting the question as asking for the solid with the *least* volume. **Incorrect Response: C.** This response could stem from a misconception that the radius of the cylinder is x, rather than $\frac{x}{2}$. **Incorrect Response: D.** This response may be the result of misinterpreting the value of $\sqrt{3}$, or another misconception related to properties of 30-60-90 triangles.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
55		D	Objective 009 **Correct Response: D.** Using a base radius of 5 cm (centimeters) and a height of 15 cm (centimeters), the volume of the can is $25\pi(15) \approx 1178.09$ cm (centimeters)^3. The maximum possible error occurs from an underestimate of the volume for the can. Assuming the can has a base radius of 5.49999 cm (centimeters) and a height of 15.49999 cm (centimeters): $(5.49999)^2\pi \cdot 15.49999 \approx 1473.01$. The difference between these values is the maximum possible error in the calculated volume: $1473.01 - 1178.09 = 294.92 \approx 294.9$ cm (centimeters)^3. **Incorrect Response: A.** This response represents the difference between the volume of a can with radius of 5.4 and a height of 15.4, $(5.4)^2\pi \cdot 15.4 \approx 1410.77$ cm (centimeters)^3, and the estimated volume of that can found by approximating its radius as 5 and its height as 15, $5^2\pi(15) \approx 1178.09$ cm (centimeters)^3. However, because the values of 5.4 and 15.4 can increase further and still round to 5 and 15, respectively, 232.7 cm (centimeters)^3 cannot be the maximum possible error. **Incorrect Response: B.** This response is the result of the misconception that an overestimate for the volume of the can will produce the maximum possible error. Using a base radius of 4.5 cm (centimeters) and a height of 14.5 cm (centimeters) for the can: $(4.5)^2\pi(14.5) \approx 922.45$ cm (centimeters)^3. The difference between the given computation and this value results in an answer of 255.6 cm (centimeters)^3. **Incorrect Response: C.** The response represents the average of the maximum value for the error calculated in response D with the underestimated value for the error calculated in response B.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
56		C	Objective 010 **Correct Response: C.** There are two cases for which one pair of rays are collinear. They are either parallel to each other or antiparallel (i.e., they line along the same line but point in opposite directions). Consider these two cases and then draw in the remaining pair of parallel rays, as shown in the following diagram.  In the top diagram, the angles are supplementary because they combine to form a straight angle. In the bottom diagram, the angles can be shown to be congruent by reasoning that the collinear rays lie along a transversal that cuts through parallel lines containing the parallel rays. **Incorrect Responses A, B, and D.** Based on the information shown above, it is clear that these statements are not true.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
57		B	Objective 010 **Correct Response: B.** The format of an indirect proof requires the assumption that the negation of the conclusion is correct. Thus, assume that $\overline{PN}$ bisects $\angle QPR$. Use this assumption and other given information to write a list of logical statements until a contradiction of either one of the given pieces of information or a theorem or definition is found. In this case, the assumption and the properties of the isosceles triangle can be used to prove that N must be the midpoint of $\overline{QR}$, which contradicts the given information about point N. **Incorrect Response: A.** This response does not match the format of an indirect proof since the assumption is not the negation of the conclusion. **Incorrect Response: C.** This response does not match the format of an indirect proof since the assumption is not the negation of the conclusion. **Incorrect Response: D.** This response assumes the negation of the conclusion, but it is not true that $\overline{QR}$ cannot be the base, so a contradiction will not be reached.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
58		A	Objective 010 **Correct Response: A.** Let x be the measure of the exterior angle. Then $6x + 12$ is the measure of the interior angle and $x + 6x + 12 = 180$. This equation yields $x = 24$, so the exterior angle measures 24°. The polygon is regular, so the exterior angles are congruent. The sum of the exterior angles of any polygon is 360°. Dividing this by 24° per angle yields 15 angles. It follows that there are 15 sides. Alternatively, one interior angle measures $6(24) + 12 = 156$ and the formula for the measure of one interior angle of a regular polygon is $m = \frac{(n - 2)180}{n}$, where n equals the number of sides. Solving $156 = \frac{(n - 2)180}{n}$ also yields $n = 15$. **Incorrect Response: B.** If the number of sides is 18, then each exterior angle measures $\frac{360}{18} = 20°$ and each interior angle measures 160°. $160 \neq 6(20) + 12$. **Incorrect Response: C.** If the number of sides is 20, then each exterior angle measures $\frac{360}{20} = 18°$ and each interior angle measures 162°. $162 \neq 6(18) + 12$. **Incorrect Response: D.** Solving the equation $x + 6x + 12 = 180$ leads to this response. However, 24 represents the measure of one of the exterior angles of the regular polygon described and not its number of sides.

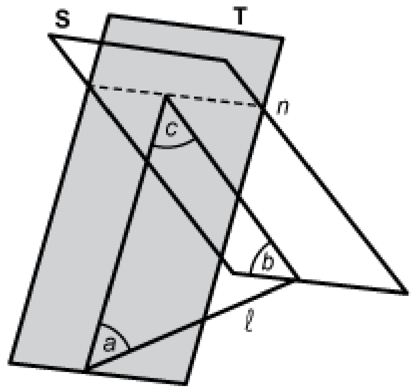
Question Number	Your Response	Correct Response	Related Objectives and Rationale
59		A	Objective 010 **Correct Response: A.** Since opposite sides of a parallelogram are congruent, they represent congruent chords of the circle and intercept congruent arcs of the circle. Each of the two distinct sets of adjacent sides intercept arcs whose sum is half the circle. The angle of the parallelogram formed by two adjacent sides is therefore inscribed in a semicircle. The measure of an inscribed angle is half the measure of its intercepted arc. Thus, the angle formed by two adjacent sides of the parallelogram is a right angle, and the inscribed parallelogram is a rectangle. **Incorrect Response: B.** It is possible to inscribe a square in a circle, but the information given is insufficient to guarantee that the inscribed parallelogram is a square. **Incorrect Response: C.** Since the inscribed parallelogram must have right angles, the rhombus would have to be a square. **Incorrect Response: D.** The inscribed parallelogram must have right angles. The only kite that is a parallelogram with right angles is a square.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
60		B	Objective 010 **Correct Response: B.** In $\triangle ABC$, use $\overline{AC}$ as the base and draw height h from point B to $\overline{AC}$ or, in the case that $\angle A$ is obtuse, to the extension of base $\overline{AC}$. Then Area = $\frac{1}{2}(AC)h$. Sin $A = \frac{h}{AB}$, so $h = (AB)\sin A$. Substituting for h in Area = $\frac{1}{2}(AC)h$ yields Area = $\frac{1}{2}(AB)(AC)\sin A$. The sines of angles between 90° and 180° have the same values as their reference angles less than 90°, so this formula is true for any triangle. **Incorrect Response: A.** The ratio is inverted. sin $A = \frac{h}{AB}$, not $\frac{AB}{h}$. **Incorrect Response: C.** While the formula is true for right triangles, it is also true for non-right triangles. **Incorrect Response: D.** The sines of angles between 90° and 180° have the same values as their reference angles less than 90°, so this formula is true for any triangle.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
61		C	Objective 010 **Correct Response: C.** To inscribe a circle in a triangle, the center of the circle must be located so that the distance from the center of the circle to each side of the triangle is the same. Since any point on an angle bisector is equidistant from the sides of the angle, the point where the three angle bisectors of the triangle intersect is equidistant from all three sides. Thus, it is the center of the inscribed circle. **Incorrect Response: A.** The altitudes of a triangle do intersect, sometimes outside the triangle or on the triangle, depending on its shape. The point of intersection is not equidistant from the sides unless the triangle is equilateral. **Incorrect Response: B.** The medians of a triangle intersect at a point called the centroid, which is the point about which the mass of the triangle is equally distributed. The centroid is not equidistant from the sides unless the triangle is equilateral. **Incorrect Response: D.** The perpendicular bisectors of the sides intersect at a point that is the center of the circle that can be circumscribed about the triangle, since each point on the perpendicular bisector of a side is equidistant from the endpoint of that side.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
62		C	Objective 011

Correct Response: C. Consider how the information in the diagram supports a proof by contradiction.



Begin by assuming line ℓ is perpendicular to planes S and T and that these planes are not parallel. Suppose a triangle is formed from the two points where line ℓ intersects the planes and a point along the line of intersection between the planes, such as the one shown in the diagram. This triangle would have two perpendicular angles, which is impossible. The assumption that the planes are not parallel is incorrect. Therefore, the planes must be parallel.

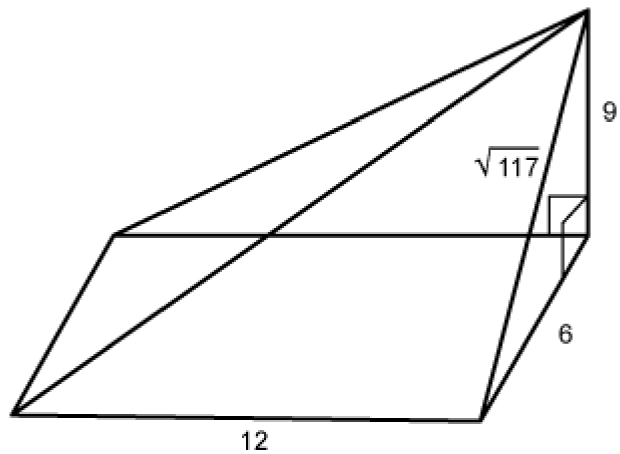
Incorrect Response: A. The given line will not be perpendicular to lines in the plane that do not pass through the point of intersection of the given line and the plane.

Incorrect Response: B. In 3-dimensional space, two lines perpendicular to the same line may be skew to each other since they do not have to lie on the same plane.

Incorrect Response: D. A counterexample is given by considering three intersecting faces of a cube. Two vertical faces are each perpendicular to the base but intersect each other.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
63		B	Objective 011

Correct Response: B. This net folds to create a rectangular pyramid whose vertex is above the point where the 12, 6, and 9 unit segments intersect—directly above one of the corners of its rectangular base. The base is a 12 by 6 rectangle, and the height is 9. The volume is $\frac{1}{3}(12 \cdot 6 \cdot 9) = 216$ cubic units.



Incorrect Response: A. This response may be the result of using the right triangle whose legs measure 12 and $\sqrt{117}$ as the base of the pyramid and 6 as its height:

$$\frac{1}{3}\left(\frac{12\sqrt{117}}{2} \cdot 6\right) = 12\sqrt{117}$$

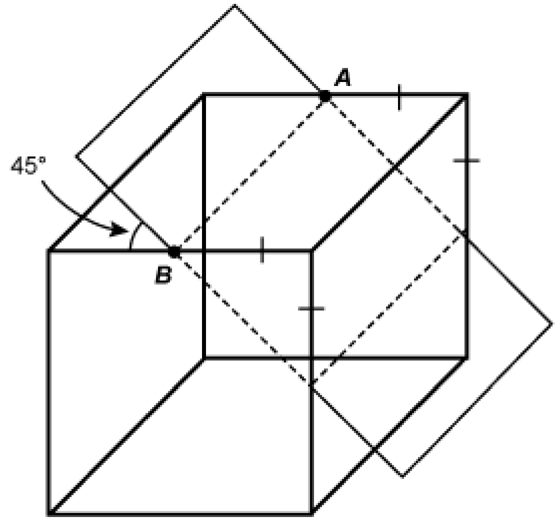
cubic units.

Incorrect Response: C. This response may be the result of using the rectangle as the base, but incorrectly using $4 \times 2 \times \frac{1}{2} \times \ell \times w$ as the height: $\frac{1}{3}(12 \cdot 6 \cdot \sqrt{117})$ $= 24\sqrt{117}$ cubic units.

Incorrect Response: D. This response may be the result of using the right triangle whose legs measure 12 and 9 as the base and 6 as the height and failing to recognize that the solid is a pyramid: $\frac{1}{2}(12 \cdot 9) \cdot 6 = 324$ cubic units.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
64		C	Objective 011

Correct Response: C.



The plane forms two pairs of parallel edges when it cuts through the top face of the cube at a 45° angle. The pair of edges formed along the top and right surfaces are congruent because they are both equal to the length of the cube. The pair of edges along the front and rear faces are also congruent. They can be thought of as the hypotenuses of the two isosceles right triangles, as shown in the diagram. Both pairs of adjacent sides are located within perpendicular planes (i.e., the adjacent faces of the cube) and so the angle of their intersection must be 90°. The quadrilateral must be a rectangle because it has two pairs of congruent sides and four right angles.

Incorrect Response: A. The quadrilateral is a parallelogram, but that is not its most descriptive name.

Incorrect Response: B. Since adjacent sides of the quadrilateral are not equal, it cannot be a rhombus.

Incorrect Response: D. Since adjacent sides of the quadrilateral are not equal, it cannot be a square.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
65		D	Objective 011 **Correct Response: D.** The plot can be divided into 2 right triangles similar to a 3-4-5 right triangle, one trapezoid, and a large triangle. Using ratios, the bases of the triangles similar to a 3-4-5 triangle are 15 yards and 12 yards. Between them is a trapezoid of height 21 yards and parallel bases 20 yards and 16 yards. The bottom triangle has base 48 yards and height 10 yards. The total area is $\frac{1}{2}(15)(20) + \frac{1}{2}(21)(20 + 16) + \frac{1}{2}(12)(16) + \frac{1}{2}(48)(10) = 864$ square yards. One square yard = 9 square feet, so 864(9) = 7776 square feet. Every 1000 square feet requires 3 pounds of seed, so $7776\left(\frac{3}{1000}\right) \approx 23.3$ pounds. **Incorrect Response: A.** This response may be the result of calculating the area correctly as 864 square yards, but multiplying by 3 rather than 9 to convert to square feet. **Incorrect Response: B.** This response may be the result of calculating the areas of the triangles correctly, but miscalculating the area of the trapezoid, then multiplying by 3 rather than 9 to convert to square feet. **Incorrect Response: C.** This response may be the result of calculating the areas of the right triangles and the trapezoid correctly, but using the wrong base length for the bottom triangle.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
66		A	Objective 011 **Correct Response: A.** If the linear scale factor is k, then the original surface area (SA) will be multiplied by k^2 and the original volume (V) by k^3. The new ratio of surface area to volume will then be $\frac{k^2(SA)}{k^3(V)} = \frac{1}{k}\left(\frac{SA}{V}\right) = \frac{1}{k} \cdot r$. This response may also be found by setting up the formulas for surface area and volume for each cylinder and using algebraic properties to reduce the respective ratios. **Incorrect Response: B.** This response could be the result of simply assuming that the scale factor will apply to the efficiency ratio as well as to the linear dimensions. **Incorrect Response: C.** This response could be the result of understanding that the volume scale factor will be k^3 and misunderstanding the ratio components. **Incorrect Response: D.** This response could be the result of understanding that the volume scale factor will be k^3 and assuming that the original efficiency ratio will be multiplied by the volume scale factor.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
67		D	Objective 011 **Correct Response: D.** A cross-section of the cone along its axis showing where the frustum cut was made will contain similar right triangles. Let the slant height of the small cone that was cut off be x. Then $\frac{x}{x + 13} = \frac{4}{9}$, $x = 10.4$, and the slant height of the large cone is 23.4. Using the Pythagorean Theorem, $9^2 + h^2 = 23.4^2$ and $h = 21.6$. **Incorrect Response: A.** This response could be the result of using the incorrect proportion $\frac{x}{13} = \frac{4}{9}$, to get $x \approx 5.8$, then $18.8^2 = 9^2 + h^2$ to get $h = 16.5$. **Incorrect Response: B.** This response could be the result of drawing the height of the frustum from the edge of the upper surface to find a 5, 12, 13 right triangle, so the height of the frustum is 12 units. Let y be the height of the small cone cut off. The incorrect proportion $\frac{y}{12} = \frac{4}{9}$ yields $y \approx 5.3$, so the total height is $12 + 5.3 = 17.3$. **Incorrect Response: C.** This response could be the result of using the incorrect proportion $\frac{x}{13} = \frac{4}{9}$, to get $x \approx 5.8$, then incorrectly setting up $18.8^2 + 9^2 = h^2$ to get $h = 20.8$.

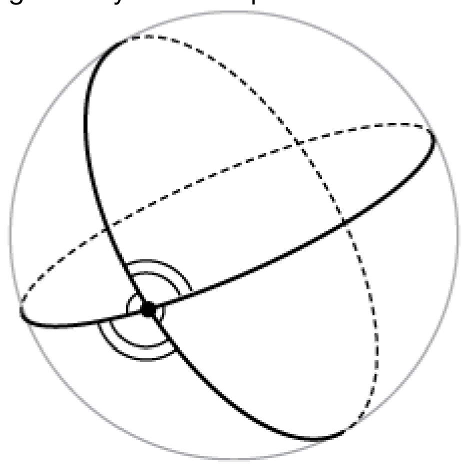
Question Number	Your Response	Correct Response	Related Objectives and Rationale
68		D	Objective 012 **Correct Response: D.** The equation provided describes a circle with a center of (0, 4) and a radius of 5. Substituting 0 for x gives the y-intercepts for the circle: $(y - 4)^2 = 25 \rightarrow y - 4 = \pm 5 \rightarrow y = 4 \pm 5 \rightarrow y = 9, -1$. The parabola intersects the y-axis in two places, so x is a function of y in its equation. The parabola will intersect 9 and –1 on the y-axis if it has factors $(y - 9)(y + 1) = y^2 - 8x - 9$. When $y = 0$, the parabola must have a value of 1, so $x = a(y^2 - 8y - 9) \rightarrow 1 = a((0)^2 - 8(0) - 9) \rightarrow 1 = -9a \rightarrow a = -\frac{1}{9}$. This gives the equation $x = -\frac{1}{9}(y^2 - 8y - 9)$. **Incorrect Response: A.** This equation may be the result of assuming that the parabola must have the same x-intercepts as the circle provided. **Incorrect Response: B.** This equation is the result of the misconception that the parabola has x-intercepts that are the same as the y-intercepts for the circle provided. **Incorrect Response: C.** This equation may be the result of using the x-intercepts for the circle instead of the y-intercepts.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
69		C	Objective 012 **Correct Response: C.** It is necessary to find the center and semi-axis lengths of the ellipse. Use completing the square. The equation given can be rewritten as $4(x^2 + 4x + 4) + 9(y^2 - 2y + 1) = 11 + 16 + 9$ or $4(x + 2)^2 + 9(y - 1)^2 = 36$. Dividing by 36 yields $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$. The center of the ellipse is (–2, 1) . The major axis is horizontal with half-length 3. The minor axis is vertical with half-length 2. The distance from the center to one focus is $\sqrt{3^2 - 2^2}$, or $\sqrt{5}$. The foci are located on the major axis of the ellipse. Thus, the foci are $\sqrt{5}$ units to the right and left of the center of the ellipse, at *x*-coordinates of $-2 \pm \sqrt{5}$. **Incorrect Response: A.** This response is the result of calculating the distance from the center to one focus correctly, but placing the foci on the vertical axis rather than on the horizontal axis. **Incorrect Response: B.** This response is the result of calculating the distance from the center to one focus using $\sqrt{3^2 + 2^2}$ and then placing the foci on the vertical axis rather than on the horizontal axis. **Incorrect Response: D.** This response is the result of calculating the distance from the center to one focus using $\sqrt{3^2 + 2^2}$ and then placing the foci on the horizontal axis.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
70		A	Objective 012 **Correct Response: A.** The slope of segment *AB* is $\frac{3-(-7)}{5-1}=\frac{5}{2}$. The slope of a line perpendicular to segment *AB* is the opposite reciprocal of $\frac{5}{2}$, which is $-\frac{2}{5}$. The midpoint of segment *AB* is $\left(\frac{5+1}{2}, \frac{3+-7}{2}\right)$ or (3, –2). Using point-slope form, the equation of the perpendicular bisector is $y+2=-\frac{2}{5}(x-3)$ or $2x+5y=-4$. **Incorrect Response: B.** This response may be the result of calculating the slope correctly but thinking that the midpoint is found using $\left(\frac{5-1}{2}, \frac{3-(-7)}{2}\right)$ = (2, 5) and not checking the validity of this result. The equation thus becomes $y-5=-\frac{2}{5}(x-2)$ or $2x+5y=29$. **Incorrect Response: C.** This response may be the result of calculating the midpoint correctly but thinking that the slope of segment *AB* is $\frac{5-1}{3-(-7)}=\frac{2}{5}$. Then the negative reciprocal is $-\frac{5}{2}$ and the equation becomes $y+2=-\frac{5}{2}(x-3)$ or $5x+2y=11$. **Incorrect Response: D.** This response may be the result of calculating both the midpoint and the slope incorrectly. Using (2, 5) and $-\frac{5}{2}$ results in $y-5=-\frac{5}{2}(x-2)$ or $5x+2y=20$.

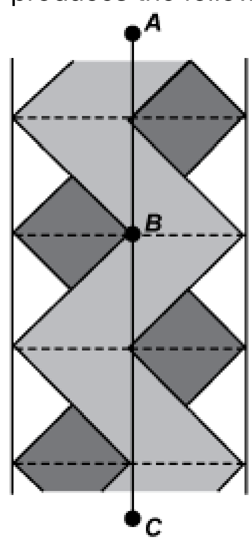
Question Number	Your Response	Correct Response	Related Objectives and Rationale
71		B	Objective 012 **Correct Response: B.** The median of a triangle connects a vertex to the midpoint of the opposite side. Find the midpoint of side *AB*: $\left(\frac{6+0}{2}, \frac{0+8}{2}, \frac{0+0}{2}\right)$ = (3, 4, 0). The length of a segment in 3-space is given by the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Thus, the length of the median is $\sqrt{(7-3)^2 + (6-4)^2 + (10-0)^2} = 2\sqrt{30}$. **Incorrect Response: A.** This response may be the result of finding the midpoint of side *AB* correctly, then interpreting the diagram incorrectly to think that point *C* is directly above the midpoint of side *AB* and subtracting only the *z*-coordinates to get the distance. **Incorrect Response: C.** This response may be the result of finding the midpoint of side *AB* correctly, subtracting corresponding coordinates to get vector (4, 2, 10), but incorrectly calculating its magnitude by adding the components: 4 + 2 + 10 = 16. **Incorrect Response: D.** This response may be the result of finding the midpoint of side *AB* correctly, but incorrectly applying the distance formula using addition rather than subtraction of the respective coordinates: $\sqrt{(7+3)^2 + (6+4)^2 + (10+0)^2} = 10\sqrt{3}$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
72		C	Objective 012 **Correct Response: C.** Using segment *BC* as the base of triangle *ABC*, its height is the distance from *A*(–6, 8) to the line *y* = 2. That distance is 6. The base length is 5 – (–4) = 9. The area of triangle *ABC* is $\frac{1}{2}(9)(6) = 27$. Triangle *A'B'C'* is similar to triangle *ABC* and has a scale factor of $\frac{4}{3}$. The ratio of the area is $\left(\frac{4}{3}\right)^2$ or $\frac{16}{9}$. The area of triangle *A'B'C'* is $\frac{16}{9}(27) = 48$. **Incorrect Response: A.** This response may be the result of calculating the area of triangle *ABC* correctly but using the scale factor rather than its square to find the area of triangle *A'B'C'*: $27\left(\frac{4}{3}\right) = 36$. **Incorrect Response: B.** This response may be the result of calculating the height of triangle *ABC* incorrectly using the length of side $AB = \sqrt{36 + 4} = 2\sqrt{10}$. The area becomes $9\sqrt{10}$ and then is multiplied by the scale factor: $9\sqrt{10}\left(\frac{4}{3}\right) = 12\sqrt{10}$. **Incorrect Response: D.** This response may be the result of calculating the height of triangle *ABC* incorrectly as in response B to get the area $9\sqrt{10}$, then correctly using the square of the scale factor: $9\sqrt{10}\left(\frac{4}{3}\right)^2 = 16\sqrt{10}$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
73		B	Objective 012 **Correct Response: B.** This axiom still holds in spherical geometry. An example is shown below.  **Incorrect Response: A.** This response is not true in spherical geometry; antipodal points like the north and south poles on a globe have many different routes between them that are of equal length. **Incorrect Response: C.** This statement is true for plane geometry, but it is not true in spherical geometry, as the sum of the interior angles in a triangle on a sphere can exceed 180 degrees. **Incorrect Response: D.** This is untrue in spherical geometry. For a given line and a point not on the line, many different lines (circles) can be drawn that contain the point and do not intersect the given line.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
74		D	Objective 012

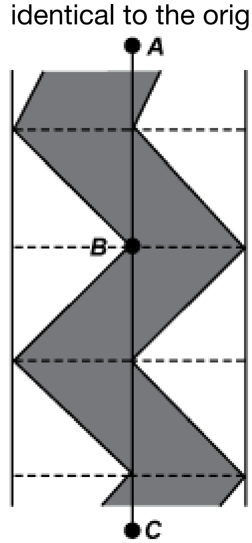
Correct Response: D. A glide-reflection across $\overline{AC}$ is a combination of a reflection over $\overline{AC}$ followed by a translation parallel to $\overline{AC}$. A reflection across the line *AC* produces the following result:



A subsequent translation up or down will produce the same pattern as the original image.

Incorrect Response: A. A dilation produces an image that is similar to the original image, but changing the size produces two non-congruent shapes.

Incorrect Response: B. A 180-degree rotation about point *B* produces the following result, which is not identical to the original image.



Question Number	Your Response	Correct Response	Related Objectives and Rationale
			Incorrect Response: C. A reflection across $\overline{AC}$ is shown in the rationale for response D. This transformation requires a translation to result in a congruent shape to the original.
75		C	Objective 013 **Correct Response: C.** The digits 0 and 1 represent 20% of the single-digit numbers. Each single-digit number represents a box of crackers. Counting the number of single digits read until a 0 or 1 is read is the same as counting the number of boxes purchased until the desired coupon is found. The average number of boxes will be the average of the results of the 100 trials. **Incorrect Response: A.** This response does not model the problem. The number 20 is only one of 90 two-digit numbers. It does not represent 20% of the two-digit numbers. **Incorrect Response: B.** This response does not model the problem. Tossing the coins 10 times models buying 10 boxes, not asking how many boxes until a coupon is found. **Incorrect Response: D.** This response does not model the problem. Tossing a coin 5 times models buying 5 boxes, not asking how many boxes until a coupon is found.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
76		B	Objective 013 **Correct Response: B.** Since the numbers picked are real numbers, there are infinitely many pairs to consider. Use an area diagram to represent the situation. Consider a square drawn on a coordinate plane with vertices (1, 1), (1, 7), (7, 1), and (7, 7). Its area is 6 • 6 = 36. The number pair must satisfy the inequality $x + y \geq 6$. This is the region in the square and above or on the line $y = -x + 6$. This line intersects the sides of the square at (1, 5) and (5, 1). The area in the square and below $y = -x + 6$ is $\frac{1}{2}(4 \cdot 4) = 8$, so the area above this line is 36 – 8 = 28. The probability that $x + y \geq 6$ is $\frac{28}{36}$ or $\frac{7}{9}$. **Incorrect Response: A.** This response satisfies the probability that $x + y \leq 6$. **Incorrect Response: C.** This response is the probability that $x + y < 6$ if x and y are whole numbers. There will be 7 • 7 pairs and 10 pairs will have a sum less than 6. **Incorrect Response: D.** This response is the probability that $x + y \geq 6$ if x and y are whole numbers. There will be 7 • 7 pairs, and 39 pairs will have a sum greater than or equal to 6.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
77		C	Objective 013 **Correct Response: C.** There are 3 branches of the tree to consider: *WW*, *WL*, and *LW*. The probability of *WW* is $\frac{1}{3} \cdot \frac{1}{5}$, of *WL* is $\frac{1}{3} \cdot \frac{4}{5}$, and of *LW* is $\frac{2}{3} \cdot \frac{1}{5}$. The sum of these probabilities is $\frac{7}{15}$. **Incorrect Response: A.** This response considers only the branches that are labeled wins: $\frac{1}{3} \cdot \frac{1}{5} + \frac{1}{5} = \frac{4}{15}$. **Incorrect Response: B.** This response is the calculation for exactly one win: $\frac{1}{3} \cdot \frac{4}{5} + \frac{2}{3} \cdot \frac{1}{5} = \frac{2}{5}$. **Incorrect Response: D.** This response is the result of thinking that P(2 wins) + P(win first) + P(win second) = $\frac{1}{3} \cdot \frac{1}{5} + \frac{1}{3} + \frac{1}{5} = \frac{3}{5}$.
78		A	Objective 013 **Correct Response: A.** The probability of a person winning on any particular day is $\frac{1}{150}$. For the entire week, the expected value of winning is $\frac{1}{150}$(40 + 50 + 20 + 20 + 20 + 50 + 100) = \$2.00. For Friday and Saturday, the expected value of winning is $\frac{1}{150}$(50 + 100) = \$1.00. Thus, the difference is \$1.00. **Incorrect Response: B.** This response is the expected value of winnings for the entire week. **Incorrect Response: C.** This response is the difference between the winning amounts on Friday and Sunday. **Incorrect Response: D.** This response is the sum of the winnings for the week minus the sum of the winnings through Thursday.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
79		B	Objective 013 **Correct Response: B.** The number of songs that comes from the second album is unknown but can be determined from the equation $\frac{2}{20} \cdot \frac{x}{20} = \frac{3}{100} \rightarrow x = 6$. A total of 8 songs from the first two albums appears on the greatest hits album. The fraction $\frac{8}{20}$ reduces to $\frac{2}{5}$. **Incorrect Response: A.** The number of songs on album 2 is 6, and $\frac{6}{20}$ simplifies to this result. **Incorrect Response: C.** On the greatest hits album, $\frac{2}{20}$ songs are from album 1. If $\frac{1}{2}$ of the songs (10 songs) comes from the first two albums, then 8 songs must be from album 2. However, $\frac{2}{20} \cdot \frac{8}{20} = \frac{16}{400} = 4\%$, not the 3% specified in the description. **Incorrect Response: D.** This result would be obtained if the order in which the songs were played did not matter.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
80		D	Objective 014 **Correct Response: D.** This sampling method is similar to stratified random sampling. This method ensures that caregivers at each school have an opportunity to be included in the sample. Calling non-respondents from this sample should increase the response rate and make it more representative of the overall population. This method is most effective when all 30 caregivers respond and the number of caregivers at each school is equal. **Incorrect Response: A.** This is an attempted census. The non-response rate will contribute to a bias. Those who do respond may have a different opinion than those who do not. **Incorrect Response: B.** This is cluster sampling. The opinions of the parents or guardians from one school may not be representative of the opinions of those from the other schools. **Incorrect Response: C.** This asks for a voluntary response from those who have access to a computer. Only those who have internet and feel strongly will respond.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
81		B	Objective 014 **Correct Response: B.** A box-and-whisker plot displays the lowest value, first quartile value, median, third quartile value, and highest value of a data set. The stem-and-leaf plot already orders the data. Since there are 25 values, the median is the 13th value, which is 25. The first quartile value is the median of the lowest 12 values, or 12. The third quartile value is the median of the highest 12 values, or 28. The first whisker extends from 2 to 12. The box extends from 12 to 28 with a vertical segment at 25. The last whisker extends from 28 to 34. Thus, B is the correct box plot. **Incorrect Response: A.** This response has the correct median, but incorrect first and third quartile values. It appears that the value occurring the most often in the first 12 and last 12 data were used rather than the quartile values for the ends of the box. **Incorrect Response: C.** This response marks the median as 20, which is the mean of the data. It appears that the first and third quartiles are correct. **Incorrect Response: D.** This response marks the median as 20, which is the mean of the data. It appears that the value occurring the most often in the first 12 and last 12 data were used rather than the quartile values for the ends of the box.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
82		B	Objective 014 **Correct Response: B.** Doubling the values of the data set results in a horizontal dilation of the *x*-axis, stretching the shape of the distribution wider. The mean and median are measures of center in a data set. These measures will double when the values double. **Incorrect Response: A.** The median describes the center of a sorted data set, so if a data set is doubled, the median is doubled as well. Doubling the values of the data set results in a horizontal dilation of the *x*-axis, stretching the shape of the distribution wider. **Incorrect Response: C.** The mean describes the average value in a data set: $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$. If a data set is doubled, the median is doubled as well: $\sum_{i=1}^{n} \frac{2x_i}{n} = 2\sum_{i=1}^{n} \frac{x_i}{n} = 2\bar{x}$. Doubling the values of the data set dilates the values on the *x*-axis by a factor of 2, so the shape of the new data distribution will be twice as wide as the original shape. **Incorrect Response: D.** This conclusion is only true in the case where both the mean and the median of the original data set are equal to 0.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
83		D	Objective 014 **Correct Response: D.** The number of cars sold each day from Monday to Friday in this bar graph can be represented as the set 0, 1, 3, 4, 2. The set can be ordered to find the median, which is 2: 0, 1, 2, 3, 4. The mean is $\frac{0+1+2+3+4}{5} = \frac{10}{5} = 2$, so the median and the mean are equal. **Incorrect Response: A.** This bar graph can be represented by the set 2, 4, 4, 4, 2. The sorted values of this set are 2, 2, 4, 4, 4. The median is 4, and the mean is $\frac{16}{5} = 3.2$. **Incorrect Response: B.** This bar graph can be represented by the set 2, 3, 4, 3, 2. The sorted values of this set are 2, 2, 3, 3, 4. The median is 3, and the mean is $\frac{14}{5} = 2.8$. **Incorrect Response: C.** This bar graph can be represented by the set 5, 4, 3, 4, 5. The sorted values of this set are 3, 4, 4, 5, 5. The median is 4, and the mean is $\frac{21}{5} = 4.2$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
84		A	Objective 014 **Correct Response: A.** The value *r* is the correlation between the *y*-values predicted by the regression equation and the observed values for *y*. Since the *r* value for Town A is greater than the *r* value for Town B, the regression equation for Town A is a better predictor of *y*-values (population) than the equation for Town B. **Incorrect Response: B.** This conclusion stems from the misconception that the *y*-intercept of both equations (the *b* value) indicates the initial population for each town. The actual population for either town could have been greater or less than the value predicted by the linear regression, so this conclusion cannot be reached from the information provided. **Incorrect Response: C.** The growth rate, *a*, indicates that the population of Town A is increasing at a rate of ≈ 0.47(1000) = 470 people per year, while the rate of change for Town B is an additional ≈ 3.86(1000) = 3860 people per year. **Incorrect Response: D.** The students have created a linear model of the data, meaning that a "line of best fit" was calculated from the data. The provided information offers insight only into the parameters of the line and the extent to which the populations of the two towns exhibit a linear association. Acceleration and deceleration are concepts that involve varying rates of change, which is information not captured by the output of linear regression model.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
85		A	Objective 015 **Correct Response: A.** Since direct substitution leads to the indeterminate form $\frac{0}{0}$, it is necessary to rewrite $\lim_{x \to 1} \frac{1 - x}{x^2 - 1}$ as $\lim_{x \to 1} \frac{-1(x - 1)}{(x + 1)(x - 1)}$. For $x \neq 1$, this expression is equivalent to $\lim_{x \to 1} \frac{-1}{(x + 1)}$. For x values close to 1, $\frac{-1}{(x + 1)}$ is close to $-\frac{1}{2}$, thus $\lim_{x \to 1} \frac{1 - x}{x^2 - 1} = -\frac{1}{2}$. **Incorrect Response: B.** This response could be the result of thinking that $\frac{0}{0} = 0$. **Incorrect Response: C.** This response could be the result of thinking that $\frac{0}{0} = 1$. **Incorrect Response: D.** This response could be the result of thinking that since $\frac{0}{0}$ is indeterminate, the limit does not exist.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
86		A	Objective 015 **Correct Response: A.** The slope of the tangent line at $x = 1$ is found by evaluating the derivative of the function for $x = 1$. Using the chain rule, $\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)$. For $x = 1$, $\frac{dy}{dx} = \frac{\sqrt{2}}{2}$ and $y = \sqrt{2}$, so the equation of the tangent line is $y - \sqrt{2} = \frac{\sqrt{2}}{2}(x-1)$ or $y = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}$. **Incorrect Response: B.** This response could be the result of neglecting to use the chain rule so $\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}$ and $\frac{1}{2\sqrt{2}}$ is used as the slope of the line. **Incorrect Response: C.** This response could be the result of correctly finding the derivative and the slope but using $x + 1$ instead of $x - 1$ in the point-slope form of the line. **Incorrect Response: D.** This response could be the result of both neglecting to use the chain rule and using $x + 1$ instead of $x - 1$ in the point-slope form of the line.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
87		D	Objective 015 **Correct Response: D.** A point of inflection is a point where the concavity of the graph changes. If there is a point of inflection, it can only occur where the second derivative is zero or is undefined. The first two derivatives of $f(x) = 3x^{\frac{2}{3}} - 2x$ are $f'(x) = 2x^{-\frac{1}{3}} - 2$ and $f''(x) = -\frac{2}{3}x^{-\frac{4}{3}} = \frac{-2}{3\sqrt[3]{x^4}}$. The second derivative of this function is undefined at $x = 0$. However, $f''(x)$ is always negative and, because it never changes sign, its graph is always concave down. Therefore, there is no point of inflection. **Incorrect Response: A.** This response could be the result of using the first derivative rather than the second derivative, setting it equal to zero and interpreting the negative exponent incorrectly as a negative solution to $x^{-\frac{1}{3}} = 1$. Neither the sign of the derivative nor the concavity changes at $x = -1$. **Incorrect Response: B.** This response could be the result of taking the derivatives correctly, identifying $x = 0$ as the value for which $f''(x)$ is undefined, but not checking to determine change in concavity. **Incorrect Response: C.** This response could be the result of using the first derivative rather than the second derivative, setting it equal to zero and solving to get $x = 1$. The slope changes at $x = 1$; the concavity does not.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
88		B	Objective 015 **Correct Response: B.** The radius expands at a rate of 9 inches per second, which is equivalent to a rate of 0.75 feet per second. The function $A(t) = \pi r(t)^2$ describes how the area of this circle changes over time. The rate at which the area changes is equal to $A'(t) = 2\pi r(t) \cdot r'(t) = 2\pi(0.75t) \cdot 0.75$. At 2 seconds, $A'(2) = 7.1$ feet per second. **Incorrect Response: A.** This response represents the radius of the circular ripple in feet per second at $t = 2$ seconds. **Incorrect Response: C.** An incorrect application of the chain rule may lead to this result. If the derivative is taken to be $A'(t) = 2\pi(0.75t)$, then $A'(2) = 2\pi(0.75 \cdot 2) \approx 9.4$. **Incorrect Response: D.** An incorrect derivative may lead to this result. If the derivative is taken to be $A'(t) = 2\pi(2 \cdot 0.75t)$, then $A'(2) \approx 18.9$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
89		C	Objective 015 **Correct Response: C.** A function is continuous for all real numbers if no real numbers are excluded from the domain of $f(x)$ and the values of the function transition smoothly at every point (i.e., there are no jumps, vertical asymptotes, or point discontinuities—the limit of the function is defined at and around every real number value). The functions $x^2 + 2$ and $x + 2$ are both continuous. Crucially, both functions return the same value for $x = 1$, and the piecewise function $f(x)$ transitions smoothly for a given interval around this point. **Incorrect Response: A.** This rational function simplifies to $f(x) = \frac{x^2 - 9}{x - 3} = \frac{(x + 3)(x - 3)}{x - 3} = x + 3$, but this function has a point discontinuity at $x = 3$. This function is not continuous because this value is excluded from its domain. **Incorrect Response: B.** This function is not defined for $x < 3$. **Incorrect Response: D.** The graph of this function jumps at the point $x = 1$. At $x = 1$, $f(1) = 1 + 2 = 3$. At $x = 1 + \varepsilon$, where ε is a small positive number, $f(1 + \varepsilon) = 1 + \varepsilon - 2 = -1 + \varepsilon$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
90		A	Objective 015 **Correct Response: A.** If $a(x) = x^5$ and $b(x) = 2x^4 + 3$, then $f(x) = a(b(x))$. By the chain rule, the derivative of $f(x) = a'(b(x)) \cdot b'(x)$. That is, the derivative is first applied to the function $a(x)$ and then x is substituted with $2x^4 + 3$. This quantity is then multiplied by $b'(x)$: $f'(x) = 5 \cdot (2x^4 + 3)^4 \cdot \frac{d(2x^4 + 3)}{dx} = 5 \cdot (2x^4 + 3)^4 \cdot (4 \cdot 2x^3)$. **Incorrect Response: B.** This response could be the result of misapplying the chain rule. In this case, the derivative for $2x^4$ multiplies the original function. **Incorrect Response: C.** This response does not apply the chain rule. **Incorrect Response: D.** This response applies the derivative directly to the term $2x^4$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
91		D	Objective 016 **Correct Response: D.** The function $G(x) = \int_{-4}^{x} f(t)\,dt$ represents net area between the graph of *f* and the *x*-axis over intervals from –4 to values of *x* in [–6, 6]. Since $\int_{-4}^{-6} f(t)\,dt = -\int_{-6}^{-4} f(t)\,dt$, the integral is positive even though the function is negative for these values of *t*. *G*(–6) = 3. (This value is computed geometrically by finding the area of a triangle with a height of 3 and a base of 2.) The value of the integral decreases as *x* increases toward –4 until it becomes *G*(–4) = 0. *G*(*x*) increases as *x* increases from –4 to 0 until it becomes *G*(0) = 4. To the right of this point, the area of the semicircle will be subtracted since it is below the *x*-axis, so *G*(*x*) will decrease as *x* increases from 0 to 4 until it becomes *G*(4) = 4 – 2π. For x > 4, area is above the *x*-axis and added, so *G*(*x*) will increase. Thus *G*(*x*) achieves its minimum value when *x* = 4. **Incorrect Response: A.** This response could result from thinking that the minimum value of *f* will also result in the minimum value of *G*. **Incorrect Response: B.** This response could result from reasoning that *G*(–6) = 3 and *G*(–4) = 0, then reasoning that for x > –4, the area will increase. **Incorrect Response: C.** This response could result from recognizing that *f* has a local minimum at *x* = 2 and thinking that *G* will have a minimum there also.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
92		C	Objective 016 **Correct Response: C.** Let $u = x^3 + 1$ so that $du = 3x^2\,dx$ and $x^2\,dx = \frac{1}{3}du$. This substitution changes the limits of the integral. For $x = 0$, $u = 1$ and for $x = 1$, $u = 2$. Substituting, $\int_0^1 x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int_1^2 u^{\frac{1}{2}}\,du$. Integration yields $\frac{2}{9}u^{\frac{3}{2}}\Big\|_1^2 = \frac{2}{9}\left(2\sqrt{2}-1\right)$. **Incorrect Response: A.** This response is the result of substituting the variables correctly but forgetting to change the limits of integration: $\frac{1}{3}\int_0^1 u^{\frac{1}{2}}\,du = \frac{2}{9}$. **Incorrect Response: B.** This response is the result of forgetting the $\frac{1}{3}$ when substituting and forgetting to change the limits of integration: $\int_0^1 u^{\frac{1}{2}}\,du = \frac{2}{3}$. **Incorrect Response: D.** This response is the result of remembering to change the limits of integration, but forgetting the $\frac{1}{3}$ when substituting: $\int_1^2 u^{\frac{1}{2}}\,du = \frac{2}{3}\left(2\sqrt{2}-1\right)$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
93		B	Objective 016

Correct Response: B. Simplify:

$$\int \frac{x^2-5}{x^2}\,dx = \int\left(\frac{x^2}{x^2}-\frac{5}{x^2}\right)dx = \int(1-5x^{-2})\,dx\ .$$

The antiderivative is

$$x-5\left(\frac{x^{-1}}{-1}\right)+C = x+\frac{5}{x}+C.$$

Incorrect Response: A. This response could be the result of simplifying correctly to get $\int(1 - 5x^{-2})dx$, but not recognizing that the antiderivative of $1dx$ is x.

Incorrect Response: C. This response could be the result of not simplifying and taking the antiderivatives of the numerator and denominator independently to get

$$\frac{\frac{x^3}{3}-5x}{\frac{x^3}{3}}+C = 1-\frac{15}{x^2}+C\ .$$

.

Incorrect Response: D. This response could be the result of simplifying to get

$$\int\left(1-\frac{5}{x^2}\right)dx,$$

, but, for the fraction, taking the antiderivatives of the numerator and denominator independently to get

$$x-\frac{5x}{\left(\frac{x^3}{3}\right)}+C = x-\frac{15}{x^2}+C.$$

Question Number	Your Response	Correct Response	Related Objectives and Rationale
94		A	Objective 016 **Correct Response: A.** The function $h = x(12 - x)$ has x-intercepts at $x = 0$ and 12. The area between the curve and the x-axis is found by evaluating an integral $\int_0^{12} (12x - x^2)dx =$ $\left(6x^2 - \frac{x^3}{3}\right)\Big\rvert_0^{12} =$ $6 \cdot 144 - 576 = 288$. **Incorrect Response: B.** This response could be the result of estimating the area using the formula for the area of an ellipse: $\frac{1}{2}\pi \cdot 6 \cdot 36 \approx 339$. **Incorrect Response: C.** This response could be the result of integrating correctly, but making an error by evaluating $6x^2$ as 72^2, so $72^2 - 576 = 4608$. **Incorrect Response: D.** This response could be the result of integrating each factor to get $\frac{x^2}{2}\left(12x - \frac{x^2}{2}\right)\Big\rvert_0^{12}$ $= 72 \cdot 72 = 5184$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
95		B	Objective 016 **Correct Response: B.** A continuous function is used to describe the rate at which the capacity of the water treatment plant is growing. The total capacity (i.e., the total water treated) at the end of five years is the integral of the capacity rate $\int c(t)dt = \int_0^5 1.2e^{0.14t}dt$. This evaluates as $1.2\frac{1}{0.14}e^{0.14t}\Big\|_{t=0}^{t=5} = 8.57(e^{0.14 \cdot 5} - e^0) \approx 8.7$ million gallons. **Incorrect Response: A.** This is $c(5)$, which represents the millions of gallons per year that the water treatment plant will treat at the beginning of its 5th year. **Incorrect Response: C.** This represents $c(1) + c(2) + c(3) + c(4) + c(5)$, the sum of the capacity rates at the end of each year. This overestimates the total amount of water treated because it applies the year end capacity rate to the entire year. In contrast, the provided information describes a gradual increase in capacity rate from the beginning to the end of the year. For example, $c(0) = 1.2$ million gallons per year, $c(1) = 1.38$ million gallons per year, and these rates are connected by a continuous exponential function. **Incorrect Response: D.** If the lower bound of the integral were ignored such that $\int c(t)dt = \int 1.2e^{0.14t}dt = 1.2\frac{1}{0.14}e^{0.14t}$, then the result at $t = 5$ is $8.57e^{0.14 \cdot 5} \approx 17.3$ million gallons.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
96		D	Objective 017 **Correct Response: D.** The student must now show that the equation is true for $k + 1$ odd numbers. If there are $k + 1$ odd numbers, the last odd will have a value of $2(k + 1) - 1$ or $2k + 1$. If the sum of k odd numbers is k^2, then the sum of $k + 1$ odd numbers must be $(k + 1)^2$. Thus, the student must show that $1 + 3 + 5 + 7 + \ldots + (2k - 1) + (2k + 1) = (k + 1)^2$. **Incorrect Response: A.** This response may be the result of thinking that "the next" means add 1 to the "last," so 1 is added on each side of the equation. **Incorrect Response: B.** This response may be the result of thinking that "the next" means add 1 to the "last," so 1 is added on the left, but the $k + 1$ is substituted correctly on the right. **Incorrect Response: C.** This response uses the correct expression for the next odd number on the left, but misinterprets $(k + 1)^2$ as $k^2 + 1$.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
97		C	Objective 017 **Correct Response: C.** According to the entries in the first row, communication goes from computer *A* to computers *B*, *C*, and *E*. Thus, the directed graph has arrows from *A* to *B*, *C*, and *E*. The entries in row 2 correspond to arrows going from *B* to *A* and *D*. The 1 in row 3 corresponds to an arrow from *C* to *B*. From rows 4 and 5, the ones correspond to arrows from *D* to *A* and *C*, and from *E* to *A*. The graph in response C shows all these. **Incorrect Response: A.** This response misinterprets connections, assuming, for example, that going from *A* to *B* is the same as going from *B* to *A*. **Incorrect Response: B.** This response misinterprets connections as in response A, but also reads the information down the columns rather than across the rows. **Incorrect Response: D.** This response reverses the expected connections. The rows of the matrix establish the origin of the connection and the columns describe their destination. This may be the result of reading down the columns of the matrix rather than across the rows.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
98		C	Objective 017 **Correct Response: C.** The equations $x + y + 35 = 49$, $y + z + 27 = 48$, and $x + z + 31 = 50$ follow from the information provided. Solving simultaneous equations yields $x = 6$, $y = 8$, and $z = 13$. The total number of students who played sports is 24 + 20 + 16 + 6 + 8 + 13 + 11 = 98. Then 212 – 98 = 114 students who played no sports. **Incorrect Response: A.** This response may be the result of subtracting the sum of 50, 49, and 48 from 212. It does not take overlaps into account. **Incorrect Response: B.** This response may be the result of subtracting the sum of 50, 49, and 48 from 212, but then adding back the 11 twice since it was counted in each of the three amounts subtracted. **Incorrect Response: D.** This response may be the result of adding 50, 49, and 48, then subtracting the sum of the overlaps of 11, 20, 24, and 16. The result, 76, is then subtracted from 212.

Question Number	Your Response	Correct Response	Related Objectives and Rationale
99		A	Objective 017 **Correct Response: A.** Three dogs from a group of 6 will be chosen. There are 6 choices for the first dog, 5 for the second, and 4 for the third. Thus, there are 6 • 5 • 4 = 120 unique orders in which 3 dogs may be selected from a group of 6 (i.e., the permutation ${}_6P_3$). Because the order of selection does not matter in this situation, divide 120 by the number of ways the three selections could have occurred (i.e., 3! = 3 • 2 • 1 = 6) to obtain 20. This same reasoning applies to choosing 3 cats from the 8 available cats: $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56 = {}_8C_3$. The number of ways to choose both cats and dogs is 20 • 56 = 1,120. **Incorrect Response: B.** This response erroneously adjusts for the number of orderings by dividing each of 120 and 56 by 3. There are 3! orders in which the selection could have occurred: {123, 231, 312, 321, 213, 132}. **Incorrect Response: C.** This response erroneously adjusts for the number of orderings by dividing each of 120 and 336 by 2. **Incorrect Response: D.** This response is the result of not understanding that the same 3 cats or dogs can be picked in different orders. It is the product (6 • 5 • 4) • (8 • 7 • 6).

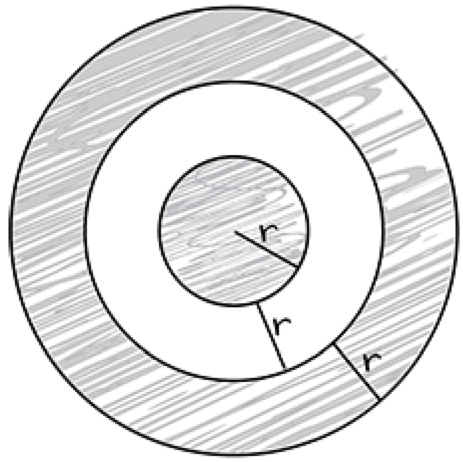
Question Number	Your Response	Correct Response	Related Objectives and Rationale
100		B	Objective 017 **Correct Response: B.** There are 12! ways to arrange 12 visitors in 12 positions and 2! ways to arrange the 2 naturalists in 2 positions. By the fundamental counting principle, there are 2! • 12! = 2 • 12! different orders in which the group may be arranged. **Incorrect Response: A.** This response represents the number of ways to arrange the visitors. However, any order for the visitors may be combined with 2 possible ways to arrange the naturalists, so this represents half of the total number of arrangements. **Incorrect Response: C.** This response describes how many ways that a group of 14 people may be arranged into a single file line. This value is too large because it counts arrangements for which the naturalists can be located in the middle of the line and visitors at the end of the line. **Incorrect Response: D.** This response combines the misconception described in response C with the application of fundamental counting principle described in response B.
Total Correct:		Review your results against the test objectives (../StudyGuide/MA_SG_obj_63.htm).	

Open Responses, Sample Responses, and Analyses

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
101	**Open Response Item Assignment #1** For each assignment, you may type your written response on the assigned topic in the box provided. **Note:** The actual test allows you to handwrite your responses on separate response sheets to be scanned for upload to the test. For this practice test, you may handwrite each response on 1–2 sheets of paper.

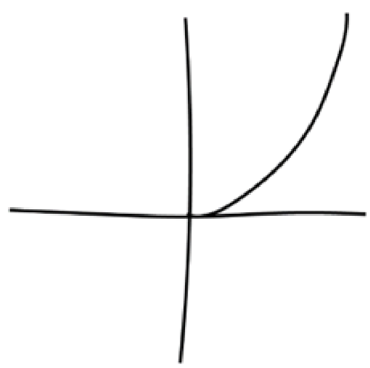
Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
First Sample Weak Response	First Sample Weak Response to Open-Response Item Assignment #1

(1)



$\pi r^2 + \pi(3r)^2 - \pi(2r)^2 = 6\pi r^2$

(2)



R	A
1	6π
2	24π
3	54π

(3) $6\pi r^2 = 150\pi$

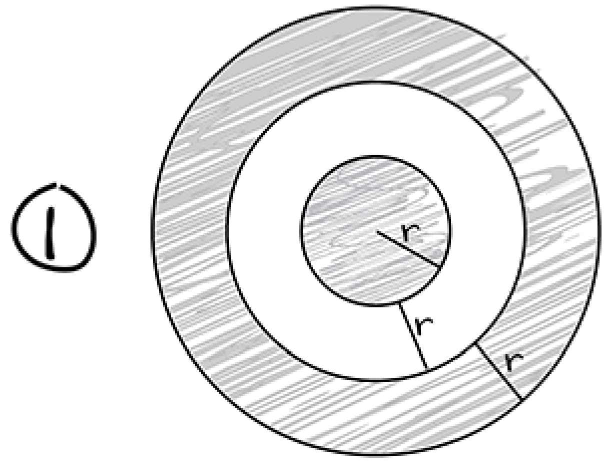
$r^2 = 25$

(4) $4\pi r^2 + 9\pi r^2 - 4\pi r^2 = 9\pi r^2$

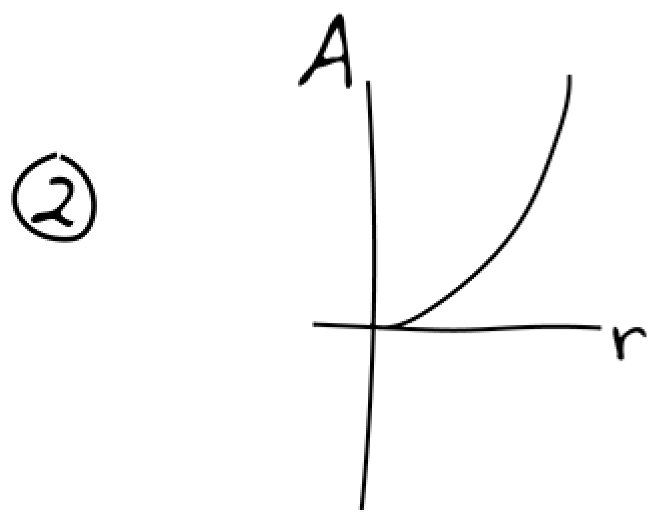
$\frac{6\pi r^2}{9\pi r^2}$

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
First Weak Response Analysis	**Analysis of First Weak Response to Open-Response Item Assignment #1** **Purpose:** All parts of the assignment are attempted but only partially achieved. For example, Part 1 is correctly solved but lacks supporting detail, explanation, and steps to reach the solution. In Part 3, the candidate attempts to address the task but does not complete it. In Part 4, it is unclear how the equation presented would find how the area of the shaded regions would change when the radius of the innermost circle is doubled. **Subject Matter Knowledge:** The candidate shows a limited understanding of what the task requires. For instance, in Part 1, the candidate uses a viable method and arrives at a correct answer, but without showing their work, there is insufficient evidence of understanding. Part 2 has a correct table of values but does not indicate how those values were obtained, such as defining the formula used to calculate A in terms of R. Additionally, the graph lacks labeled axes and defined coordinates. Part 3 correctly sets up a viable equation but does not solve it to completion to respond to the question being asked. In Part 4, the logic used is not explained and seems to reflect inaccurate subject matter knowledge. **Support:** Support is limited throughout the response. The candidate does not show or explain all the steps involved in solving the problem. Including intermediate steps would help to clarify the candidate's approach to the problem. **Rationale:** The candidate does not provide enough explanation to logically convey the reasoning behind the attempt to solve the problem. The reasoning provided is limited. For example, in Part 1, the candidate provides the final equation with no justification for the reasoning needed to reach this point in the solution. Additionally, in Part 4, it is unclear how the candidate is interpreting the prompt. This lack of clear analysis to find the solution is a problem throughout the response, resulting in large gaps in logic that ultimately make the response poorly reasoned.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Sample Weak Response	Second Sample Weak Response to Open-Response Item Assignment #1



The inner circle = πr^2
A second circle = $\pi(2r)^2 = 4\pi r^2$
The third (whole) circle = $\pi(3r)^2 = 9\pi r^2$
Area of large circle
– area of middle circle
+ area of small circle
= the area of the 2 shaded parts
$9\pi r^2 - 4\pi r^2 + \pi r^2 = 6\pi r^2$



Question Number	**Your Response** **Read about how your responses are scored and how to evaluate your practice responses**
	(3) $9\pi r^2 = 150\pi$ $9r^2 = 150$ $r^2 = \frac{150}{9}$ $r = \sqrt{\frac{150}{9}} = \frac{5\sqrt{6}}{3}$ (4) Area in the middle with r doubled is $\pi(2r)^2$ $16\pi r^2 - 9\pi r^2 + 4\pi r^2 = 11\pi r^2 = 4\pi r^2$ so area increased by $5\pi r^2$ $(11\pi r^2 - 6\pi r^2)$

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Weak Response Analysis	**Analysis of Second Weak Response to Open-Response Item Assignment #1**

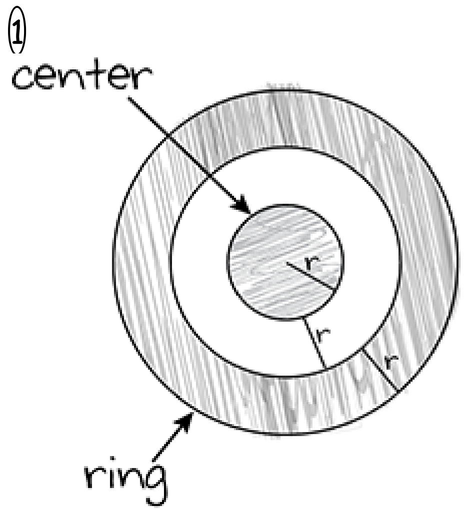
Purpose: The purpose is partially achieved. The candidate attempts to address all the components of the prompt. In Part 1, when analyzing the shaded part, each circle's area is found, and the correct process is used to find the shaded region. In Part 2, a sketch of the graph does not include all appropriate labels. The variables on the axes are not defined, and no units are given to enable reading of the graph, though the general shape of the graph shows some understanding of graphs of quadratic functions. There is no explanation of the meaning of any intercepts or extrema in the context of this situation. In Part 3, a quadratic equation is solved for *x* but there is a mistake in the original setup using $9\pi r^2$ instead of $6\pi r^2$. In Part 4, the candidate misinterprets the meaning of "double the r in the innermost circle" to mean that only the small circle gets a new radius of 2*r* when in this problem all *r*'s should become 2*r*. The response shows a lack of understanding of the meaning of ratio.

Subject Matter Knowledge: The candidate is able to utilize the correct formula for calculating the area of a circle, graphs a quadratic equation, and solves a quadratic equation. While the graph is labeled by the variables, intercepts and extrema were not labeled or explained. The candidate knows the formula for a circle and analyzes the diagram to find the shaded region correctly. In Part 3, the candidate incorrectly uses the largest circle in the calculation instead of the shaded area, but exhibits the ability to solve a quadratic equation given the context of the problem (positive solutions). In Part 4, the candidate does not realize that every *r* in the diagram should be doubled, which leads to the wrong area. The candidate does not recognize that describing the change in the areas of the shaded regions would be best represented as a multiplicative relationship that should use ratio instead of the difference in those areas.

Support: The support for this solution is limited. The response to Part 1 has reasonable support showing what the area of each circle and how they are added or subtracted to find the final area of the shaded region, but some explanation would have made it easier to follow and understand. In Part 2, there is insufficient evidence for interpreting this graph given the scenario. For Part 3, the 150π represents the shaded area; however, it is compared to the entire area of the largest circle as opposed to the shaded area found in Part 1. Without explanation of why $9\pi r^2$ was used, there is no way to know the reason for this error. In Part 4, the candidate takes the words "double the innermost *r*" to mean that 2*r* applies to the small circle only instead of understanding that every *r* now is 2*r*.

Question Number	**Your Response** **Read about how your responses are scored and how to evaluate your practice responses**
	Rationale: The response reflects limited reasoning. Even though the candidate analyzes the first part of the prompt correctly, many other errors occur. In Part 1, by finding the areas of large, middle, and small circle, the candidate analyzes the situation, explains what is happening and finds the formula. In Part 2, the graph is drawn but without identifying key features in the graph, it is unclear if its connection to the scenario is fully realized. In Part 3, it is not clear why the area of the largest circle is used instead of the shaded area in the equation to solve for the new r. In Part 4, the candidate misinterprets what double r means and does not recognize that a ratio would be a more appropriate measure of describing this change. More explanation of the reasoning used would strengthen this response.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
First Sample Strong Response	First Sample Strong Response to Open-Response Item Assignment #1

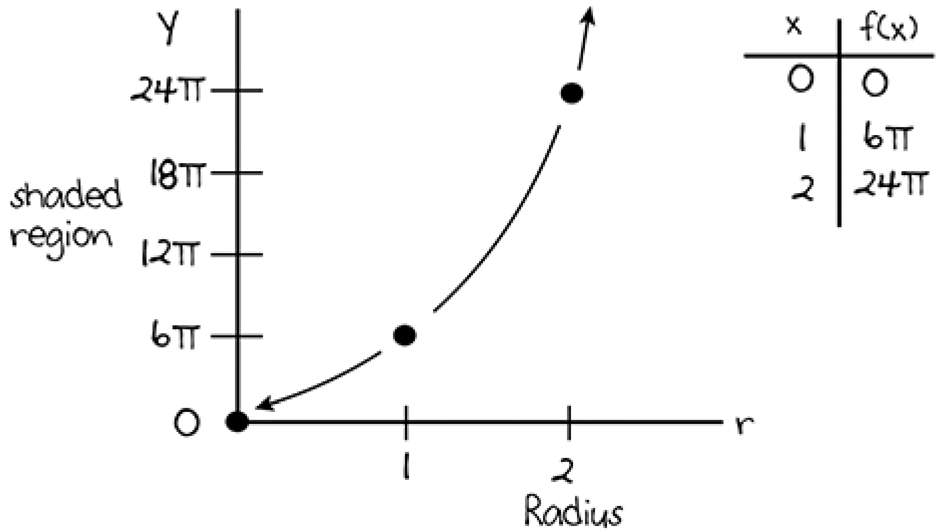


The shaded area is composed of 2 shapes. Part 1 is a "ring" and part 2 is a circle (center).
—Little circle has a radius of x.
—Middle circle has a radius of 2x.
—Large circle has a radius of 3x.
To find the area of small circle: πr^2
To find the "ring" of the large circle $(3r)^2\pi = 9r^2\pi$
And then subtract the area of the middle circle $(2r)^2\pi$
Which equals $4r^2\pi$ so ring is $9r^2\pi - 4r^2\pi = 5r^2\pi$.

Final answer for shaded region is ring area + small circle
$5r^2\pi + \pi r^2 = 6r^2\pi$ (shaded region)

(2) Graph of $f(x) = 6r^2\pi$

Question Number	**Your Response** **Read about how your responses are scored and how to evaluate your practice responses**



Since this problem is representing area, the graph is only defined in the first quadrant. The y-intercept is (0, 0) which is a circle with radius = 0 so it is just a point—no area.

(3) If shaded region is 150π, find radius

$150\pi = 6r^2\pi$

$\frac{150\pi}{6\pi} = \frac{6r^2\pi}{6\pi}$ Divide by 6π on both sides

$25 = r^2$

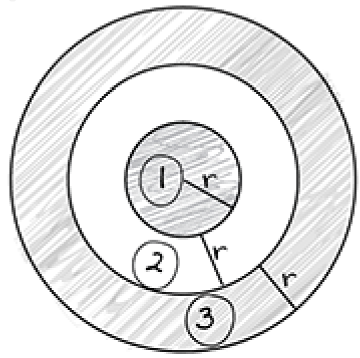
$5 = r$ (radius)

(4) If original radius doubled, then small circle area is $(2r)^2\pi$. Ring is $(6r)^2\pi - (4r)^2\pi = 36r^2\pi - 16r^2\pi = 20r^2\pi$. Shaded is $4r^2\pi + 20r^2\pi = 24r^2\pi$. It is 4 times the original. $\frac{24r^2\pi}{6r^2\pi} = 4$

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
First Strong Response Analysis	**Analysis of First Strong Response to Open-Response Item Assignment #1** **Purpose:** The candidate fully achieves the purpose of the assignment. All components of the question are fully addressed. The candidate's work is shown in detail as well as the reasoning for analyzing and solving the problem in the chosen approach. **Subject Matter Knowledge:** The candidate demonstrates substantial, accurate and appropriate subject matter knowledge. For example, the candidate selects the correct formulas and solves each formula accurately. Additionally, the candidate demonstrates knowledge of how to graph quadratic equations. The candidate also recognizes what happens when the radius is doubled and accurately represents the change in the area. **Support:** The candidate provides high-quality, relevant examples. For instance, when the area of the different circles was calculated, work is shown to justify the solution. Also, when an equation needs a solution, all the required steps are shown. The graph's axes are labeled and a table of values is shown, which aids in understanding of the graph. **Rationale:** The response shows comprehensive reasoning. For example, in Part 1, rather than simply providing the equation, the candidate demonstrates understanding of the relationship between the area of the three circles and how they affect the shaded region. Additionally, it is clear how the answers are derived in a clear and logical manner. In Part 1, the candidate explains how they are approaching the problem, labeling Part 1 as a ring and Part 2 as a circle. The candidate provides an outline of how they will solve the problem and follows through clearly, using the terms (ring, circle) that they have defined previously to explain their reasoning. Sound reasoning is further demonstrated in Part 2, where the candidate explains why the graph is only defined in the first quadrant.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Sample Strong Response	Second Sample Strong Response to Open-Response Item Assignment #1

1 See diagram.



For the formula of the shaded region, take the area of the smaller circle then the area of the larger circle minus the second circle.

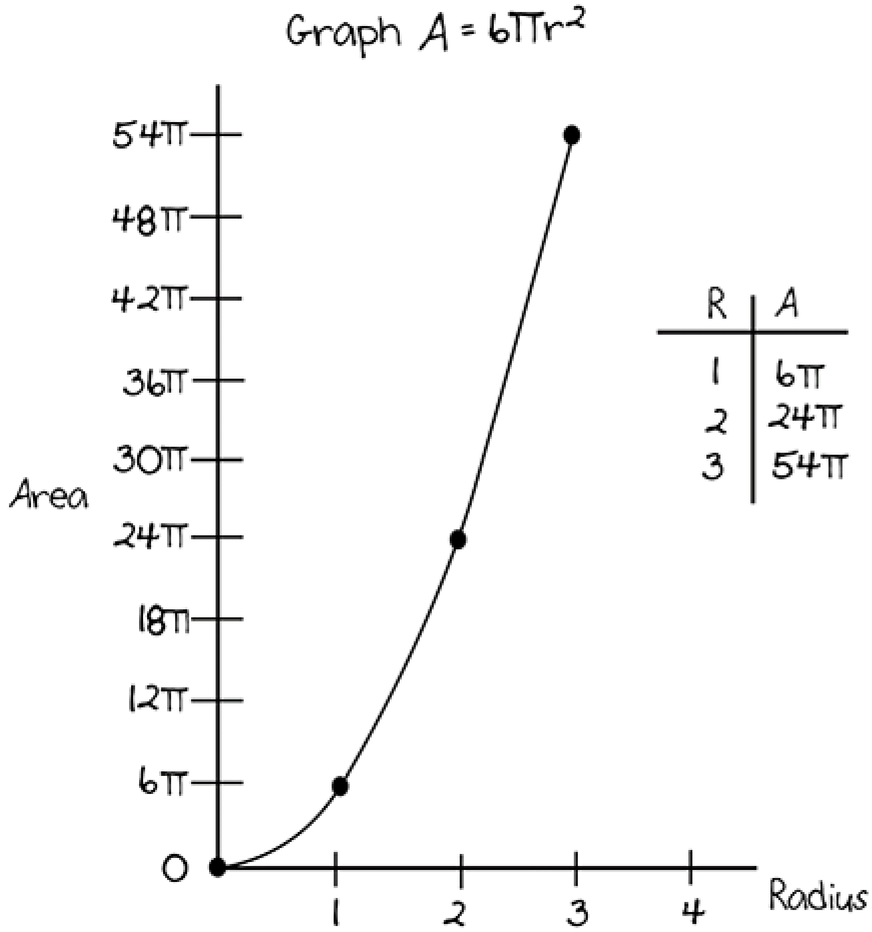
$A = \pi r^2 + \pi(3r)^2 - \pi(2r)^2$

$= \pi r^2 + 9\pi r^2 - 4\pi r^2$

$= 6\pi r^2$

2 See graph

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses



Intercept at (0, 0) represents a point.
r can equal any value greater than 0
The graph is a parabola.

(3) If Area = 150 then substitute into formula $A = 6\pi r^2$.

$150\pi = 6\pi r^2$
$25\pi = \pi r^2$
$25 = r^2$
$5 = r$

(4) If r is doubled, then radius of the first circle (innermost circle) equals 2r, the next circle is 4r and the third circle (outermost circle) is 6r.

$A = 4\pi r^2 + 36\pi r^2 - 16\pi r^2$
$= 24\pi r^2$

Compare $6\pi r^2$ to $24\pi r^2$
The area is 4 times larger.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Strong Response Analysis	**Analysis of Second Strong Response to Open-Response Item Assignment #1** **Purpose:** The candidate has fully achieved the purpose of the assignment by finding the formula in Part 1, graphing the formula, adding appropriate labels, and explaining the meaning of intercepts in Part 2, finding the radius of the innermost circle in Part 3, and finding how the relationship to the area would change if the radius were doubled in Part 4. **Subject Matter Knowledge:** The answers are accurate and use the correct equations and algebraic reasoning. The graphing is built on a table using the appropriate equations, and the graph is only drawn in the first quadrant, where it is meaningful. The candidate also demonstrates understanding of how to find the area where the radius is doubled and compare the new area with the original area. Taken as a whole, the response demonstrates substantial subject matter knowledge. **Support:** The candidate provides sound supporting evidence throughout the response by showing how they find appropriate answers to the problem. The candidate also shows their reasoning to find the shaded area through the multi-step process that was required. All the steps necessary to answer the questions are shown in a linear and clear-cut fashion. **Rationale:** The response is ably reasoned. For instance, in Part 1, the candidate shows how the answer was derived from the given information in a three-step process. In the graphing in Part 2, the candidate justifies the graph by creating a table based on the formula derived. In Part 3, the candidate explains how to find the radius given a specific area, and in Part 4, the candidate not only gives the correct answer, but explains how they reached the answer.

Question Number	Your Response **Read about how your responses are scored and how to evaluate your practice responses**
102	**Open-Response Item Assignment #2** For each assignment, you may type your written response on the assigned topic in the box provided. **Note:** The actual test allows you to handwrite your responses on separate response sheets to be scanned for upload to the test. For this practice test, you may handwrite each response on 1–2 sheets of paper.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
First Sample Weak Response	**First Sample Weak Response to Open-Response Item Assignment #2**

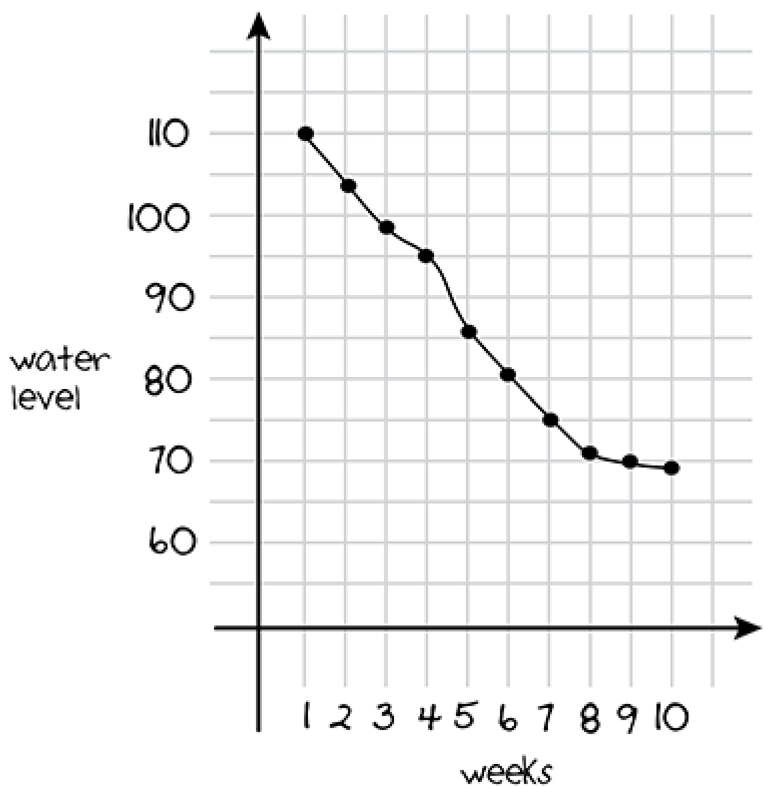
The water loss from week 1 to week 5 is 109 - 84 which is 25 inches over the 5 weeks, so the rate is 25/5 = 5 inches per week.

Divide the starting water level of 109 by 5 to get 21.8 weeks from May 18. Since only 20 weeks are troublesome (up until the beginning of October), we have enough water to last until the beginning of October and then for another week.

The water level should be always between 68 inches and 5 inches. Going down to no water is not healthy.

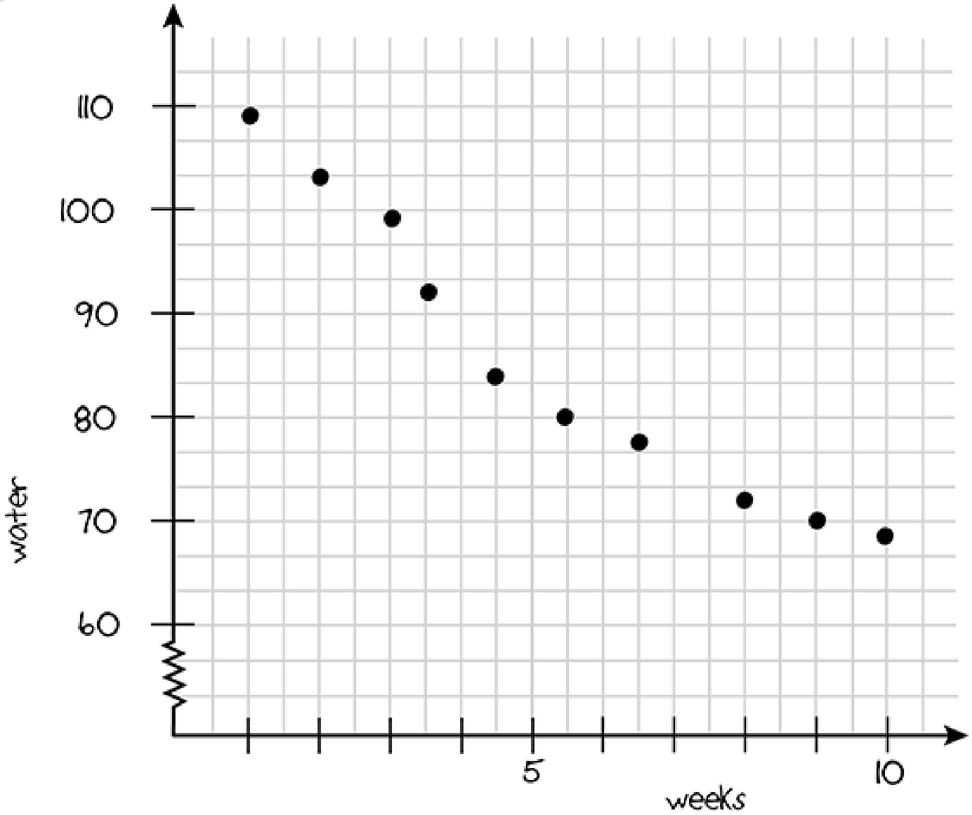
The rate of water loss could be found using a larger range of values. Instead of the first 5 values, using the first 10 could get a better rate.

Knowing the actual weather patterns in the area would make better predictions.



Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
First Weak Response Analysis	**Analysis of First Weak Response to Open-Response Item Assignment #2** **Purpose:** While all parts of the problem are addressed, they are not all answered completely, so the purpose is only partially achieved. A graph is drawn, and a rate of water loss is found. The candidate predicts when the water source would be empty if no outside water is added and gives some explanation for why the prediction may be faulty. **Subject Matter Knowledge:** There is a limited application of subject matter knowledge. Part 1 has a relatively good graph but needs the axes better defined. Units should be clearly stated. In Part 2, the candidate does find a rate of water loss accurately, but no explanation is given as to why that range was chosen. Assumptions made by the candidate are never stated or explained. The candidate never shows an understanding that water is added at some point and how that affects the prediction. There is no analysis of how the loss of water is increasing over the first 5 weeks; the rate found by the candidate is not realistic. The approach to find out when the reservoir would be empty is faulty due to the choice of rate and never considers water being added in week 5. In Part 3, a range of water heights is given but with little justification or rationale for those choices. In Part 4, two factors are given but no discussion about why they will influence the prediction. **Support:** Support is somewhat limited. The graph does show the markings on the axes and puts labels on the *x*- and *y*-axes but units are not given for water level on the *y*-axis. The candidate shows the work for how the rate of water loss is computed but provides no discussion to explain why these numbers were chosen or the possible flaws in this choice. The candidate also gives a range of heights of water but does not explain why these numbers were chosen. The influences are simply listed without any explanation of how they would affect the prediction. **Rationale:** The response reflects a limited, poorly reasoned understanding of the problem. The arithmetic for the rate of loss is partially correct (should be 4 weeks not 5, week 5 – week 1 = 4), but the candidate never recognizes that the rate of loss is increasing, and therefore the calculated rate is too low. The candidate does not clearly state or demonstrate their reasoning for the choices used to make predictions. Statements are made without explanation that would demonstrate understanding.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Sample Weak Response	**Second Sample Weak Response to Open-Response Item Assignment #2** 1. See graph. 2. From 109 to 92, the first four weeks, no supplemental water was added, so 109 - 92 = 17 and 17/4 is about 4. The rate of water is 4" a week. At ten weeks we have 68" in the reservoir so 68/4 = about 17 weeks. This means that we have more than enough water to get to October without adding anything. We only need ten more weeks of water. 3. No more water is needed. In the last ten weeks from July 27 to the beginning of October there will still be 28 inches of water in the reservoir but that doesn’t seem like it would be enough. Let’s always assume that at least 5 feet of water is in the reservoir which is 60". 60 - 28 = 32 inches of water needed to get back up to 60". I know that the supplemental adds 4" a week. 32/4 = 8 so 8 weeks of additional water will be needed. 20 - 8 = 12 weeks so at the latest water should be added starting at 12 weeks. 4. Influencing my predictions: How I chose my intervals will affect my rate. Which week I start between week 1 and week 10 will affect my decision on how long the water will last.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
	

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Weak Response Analysis	**Analysis of Second Weak Response to Open-Response Item Assignment #2**

Purpose: The purpose of the assignment is partially achieved. All parts of the prompt were attempted. The graph is labeled minimally, and points plotted relatively accurately. Work was given for the second part, finding the rate, and partially correctly done (should be 3 weeks not 4 [week 1 – 4 = 3]), but the candidate gives no explanation of why this range was chosen. The third part was answered and explained well but the assumptions made were incorrect. The last part has some possible influences that would affect the predictions but no reasoning for why these would affect the result.

Subject Matter Knowledge: The response shows knowledge of how to plot a graph though it could be more carefully plotted and axes labeled with units. The use of the first four weeks to find the rate before any supplemental water was added was a good interval to use, and finding the rate was a correct procedure, but rounding the answer added to possible inaccuracy, and it should be noted that the week span is only 3 weeks, not 4. The response did not recognize that the rate of loss was increasing by week 5, making the average computed too low. In the third part of the prompt, the response was not reasoned correctly. There was never recognition that water was supplemented starting at week five. The use of the 68, which included five weeks of supplemental water, was used to find the answer and this was never recognized, leading to an incorrect estimate. A logical reason was given for wanting a minimum of 5 feet but again not recognizing that water had already been supplemented led to another incorrect conclusion. Lastly, the influences given could be reasonable but needed to defend why these would affect the prediction.

Support: Since there was a table given, none needed to be included with their graph. Break in the y-axis was shown which shows an understanding of the whole axis not being shown. Labels should have included more information, for example, adding the words “the level of water in inches" on the y-axis would communicate more information. Support was good for finding the rate used, but no explanation was given as to why that interval was a good choice. No analysis of how the rate of loss was increasing by week 5 to a change of 8 inches was included and how that would make the rate chosen too low. No explanation was given for the choice of using 68" at week 10 as the starting point for answering the question presented but does state a conclusion based on their reasoning. Part three is supported well, giving reasons why each step is used; however, assumptions made earlier are flawed. The influences stated are possible, but support is needed to explain why these would affect the prediction and why they were chosen.

Question Number	**Your Response** **Read about how your responses are scored and how to evaluate your practice responses**
	Rationale: The overall impression of this response is that it is limited and partially correct, with poor reasoning in several parts, but there are parts that are supported and reasoned appropriately. The claim that the height of the water decreases by a rate of 4 inches per week in the first four weeks is too low. The data show the height decreases by more than 4 inches per week during this interval. In the third part, work was clearly shown and reasons given for the steps used even though the assumptions were incorrect. Though two influences were found, it would improve the response by also giving a situational influence (for example, supplemental water was an estimate and mechanical failures would affect the rate), and all influences needed explanation as to why they could affect the prediction given.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses

First Sample Strong Response

First Sample Strong Response to Open-Response Item Assignment #2

The rate of water loss is going to be based on the data from the 10-week period from May through July. I am going to use the first 4 weeks of data before they started adding 4 inches of water to get the rate of water usage. Using figures from the table, subtract 92 (level at week 4) from 109 (level at week 1), which results in 17 inches lost. Dividing 17 by 3 weeks will give us the rate of 5.67 inches per week.

At Week 10, the water level was 68, but that reflected at least 5 weeks of additional 4 inches/week of water added. On week 5 the process of adding an additional four inches was started but it would take time to have an effect, so decided not to include it in the computation. Without this addition, it would be 48 inches of water instead of 68 inches. (5 weeks x 4 inches of water added = 20 inches of additional water. 68 - 20 = 48 inches.) To predict when the reservoir would have run out of water (reached 0) if the additional reserves were not added, we would have to lose an additional 48 inches of water. 48 divided by 5.67 (the average loss) would be about 9 more weeks but it is only ten more weeks until Oct. 1 so in theory additional water would be needed.

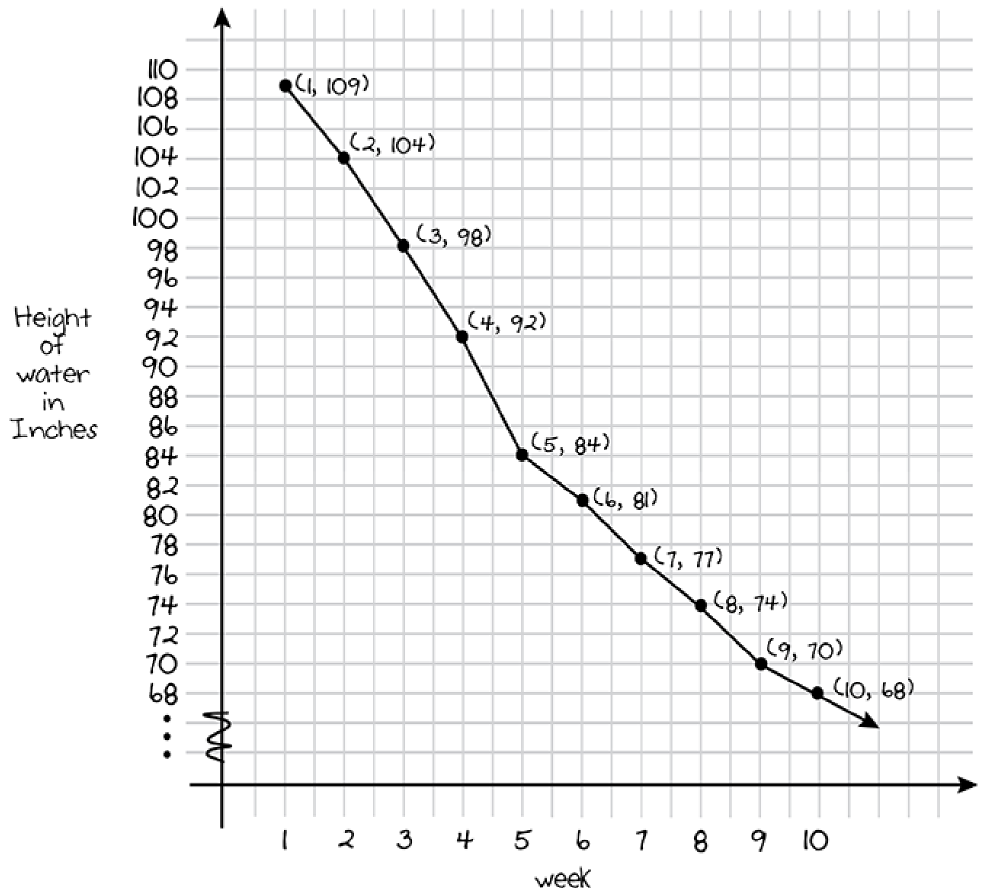
$0 < w < 68 - 5.67t$ where $0 < t < 10$. The remaining ten weeks is defined by t. The range for w was chosen because there are 68 inches of water at the end of 10 weeks for the upper limit. The lower limit should never reach zero water in the reservoir. The remaining 10 weeks until Oct. 1 are defined by t.

Reasons the calculations would be inaccurate:

If usage goes up in the last ten weeks, it affects our calculation and more water would have to be added.

If there was an unexpected large amount of rain which is atypical, then water levels would be high and additional water would not be needed.

Also, I based my prediction on the average loss during the 4 weeks in the table. If I were to base my prediction on the first 5 weeks only, my rate would be different, which would affect the prediction of when the reservoir runs dry.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
	

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
First Strong Response Analysis	**Analysis of First Strong Response to Open-Response Item Assignment #2** **Purpose:** The candidate has responded fully to all the parts of the prompt. A graph was drawn for the first question. An equation was found to predict what week there would be no more water. A range for the water level was found and explained. Things that could affect the prediction were stated and explained how they would affect the prediction. **Subject Matter Knowledge:** There is substantial, accurate and appropriate application of subject matter knowledge. The chart of values is plotted on the *x-y* graph to see the shape of the curve. The graph is carefully drawn and labeled clearly, and the scale is shown. Analysis is provided to explain how the rate of water loss was found. Assumptions that were made are explained. A range of values is computed to see when water needed to be added. Possible outside forces that might affect the predictions and what effect it would have on water levels are discussed. **Support:** Supporting evidence is found for each part. The graph is drawn with clearly defined and labeled axes, and each point on the graph has its coordinate given. Calculations for how the rate of water loss were found and the reasoning for the choices are provided. An equation for the limits on the loss of water before actions needed to be taken is developed and explained. Some possible reasons why the prediction might not be accurate are given and explained. **Rationale:** The response reflects an ably reasoned understanding of the topic. The graph reflects the relationship between different weeks and water height. The data used to find the rate of water loss is clearly shown, and reasons for using that data are explained. Each part of the prompt explains the reasoning used and shows calculations so that the process can be clearly understood.

Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Sample Strong Response	**Second Sample Strong Response to Open-Response Item Assignment #2**

1: See graph.

2: I am assuming that week 5 has not had water added – so they added water (4" per week) in weeks 6-10. At week 10 there would be 20" of water less if there was no supplement (5 weeks x 4 inches of water added = 20 inches of water) 68 - 20 = 48" at week 10 instead of 68". To find the rate of loss over the ten weeks when no supplemental water was added I will find the difference between week 1 (109") and the 48" calculated at week 10 when no extra water was added to the reservoir.

The water changed by 109 - 48 = 61" from week 1 to week 10 and at an average rate of 61/9 = 6.8 inches per week.

Assume that the reservoir can/will go down to zero then

48 - 6.8t = 0 if t = number of weeks until it is empty

48 = 6.8t and t = 7.1 weeks. The reservoir will be empty at this rate between 7 and 8 weeks after the first ten weeks, between the 17th and 18th week.

Additional water will be needed since it will be 3 more weeks before Oct. 1.

3: Assume ten more weeks until the beginning of October- Now t = the number of weeks until Oct. 1 after week 10 and w = the depth of the water at the end of each week in inches

I found the average rate of loss from week 5 through 10 when water was added each week.

84" - 68" = 16" and 16"/5 = 3.2 inches of loss each of the last five weeks in the table.

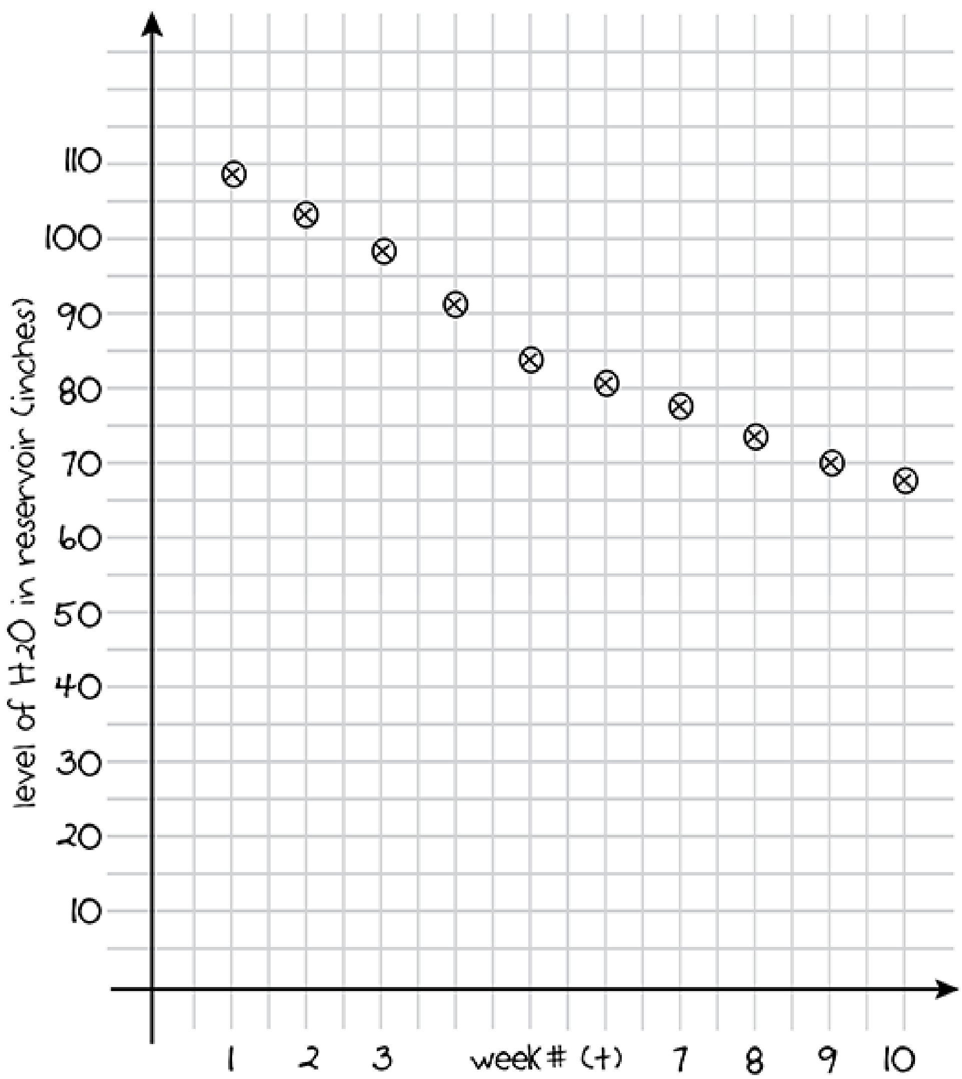
68 - 3.2t = 0, 0 < t < 10 weeks, 68 > w > 36 inches

At the end of 10 weeks 68 - 3.2(10) = 36 inches of water left in the reservoir.

4: Factors that would influence my predictions/accuracy would include:

a. My assumptions could be false. For example, letting the water go to 0 would not be logical, safe, or reasonable.
b. Assuming rainfall will continue at this constant rate

Question Number	**Your Response** **Read about how your responses are scored and how to evaluate your practice responses**
	is a big assumption since there may be a drought or an above average amount of rain and typically water usage often goes up in the hot weather. c. That the supplemental water supply can sustain this rate of 4 additional inches per week. d. That I assumed June 15th had not been supplemented by 4". e. Could have used the rate of loss over the first five weeks instead of over the ten weeks, which would have altered the average rate and therefore have changed the result/solution.



Question Number	Your Response Read about how your responses are scored and how to evaluate your practice responses
Second Strong Response Analysis	**Analysis of Second Strong Response to Open-Response Item Assignment #2** **Purpose:** The purpose of the assignment was fully achieved. Each part is thoroughly completed with calculations shown and explanation of the reasoning used. All assumptions are stated and reasoning explained. **Subject Matter Knowledge:** Part 1 shows a carefully labeled and plotted graph of the given table of water depth over a ten-week period. Part 2 provides an explanation of the assumptions that were used and resulting calculations. The result yields a well-reasoned prediction that water would need to be added before Oct. 1. Part 3 includes an equation and ranges for both t and w that are reasonable, calculated, and justified fully. Part 4 shows an understanding of the physical situation and lists many possible scenarios that would affect the prediction. **Support:** The supporting evidence is sound. Part 1 demonstrates a clear, easy-to-read and understand graph of the situation. Part 2 states assumptions and includes the reasons why those assumptions were made. The work to arrive at a conclusion is clearly shown and explained. Part 3 re-evaluates the situation with the information that there will not be enough water and re-assesses the parameters that were used, giving a reasonable new predication for the situation. Part 4 lists five plausible scenarios that could change the prediction, showing an understanding of the real-world situation given. **Rationale:** The response reflects an ably reasoned, comprehensive understanding of the topic. All parts are addressed and thoroughly reasoned with clear explanations given for assumptions used, calculations made, and conclusions formulated.

Review the Performance Characteristics and Score Scale (../StudyGuide/MA_SG_CRI_63.asp#scoring) for Written Performance Assignments.

Multiple Choice Question
Practice Test Evaluation Chart

In the evaluation chart that follows, the multiple-choice questions are arranged in numerical order and by test objective. Check your responses against the correct responses provided to determine how many questions within each objective you answered correctly.

Subarea I: Number Sense and Operations

Objective 0001: Apply knowledge of the properties and structure of the real number system.

Question Number	Your Response	Correct Response
1		B
2		B
3		A
4		C
5		C
6		C

/6

Objective 0002: Apply knowledge of the properties and structure of the complex number system and linear algebra.

Question Number	Your Response	Correct Response
7		D
8		B
9		B
10		B
11		A
12		A

/6

Subarea I (Objectives 0001–0002) Total /12

Subarea II: Relations, Functions, and Algebra

Objective 0003: Analyze and apply algebraic techniques to expressions, equations, and inequalities.

Question Number	Your Response	Correct Response
13		B
14		A
15		B
16		A
17		D
18		A

/6

Objective 0004: Apply the principles and properties of relations and functions.

Question Number	Your Response	Correct Response
19		C
20		C
21		C
22		B
23		C
24		D

/6

Objective 0005: Apply the principles and properties of linear, absolute value, and quadratic relations and functions.

Question Number	Your Response	Correct Response
25		B

Question Number	Your Response	Correct Response
26		B
27		B
28		B
29		C
30		A

/6

Objective 0006: Apply the principles and properties of exponential and logarithmic relations and functions.

Question Number	Your Response	Correct Response
31		A
32		B
33		A
34		A
35		C
36		B
37		C

/7

Objective 0007: Apply the principles and properties of polynomial, radical, and rational relations and functions.

Question Number	Your Response	Correct Response
38		B
39		D
40		D
41		D
42		C
43		B
44		C

/7

Objective 0008: Apply the principles and properties of trigonometric functions and identities.

Question Number	Your Response	Correct Response
45		A
46		A
47		B
48		B
49		C
50		B

/6

Subarea II (Objectives 0003–0008) Total /38

Subarea III: Geometry and Measurement

Objective 0009: Apply the principles, concepts, and procedures related to units and measurement.

Question Number	Your Response	Correct Response
51		C
52		C
53		C
54		B
55		D

/5

Objective 0010: Apply the axiomatic structure of Euclidean geometry.

Question Number	Your Response	Correct Response
56		C
57		B
58		A
59		A
60		B
61		C

/6

Objective 0011: Apply the principles and properties of Euclidean geometry to solve problems involving two- and three-dimensional objects.

Question Number	Your Response	Correct Response
62		C

Question Number	Your Response	Correct Response
63		B
64		C
65		D
66		A
67		D

/6

Objective 0012: Apply the principles and properties of coordinate and transformational geometry and the characteristics of non-Euclidean geometries.

Question Number	Your Response	Correct Response
68		D
69		C
70		A
71		B
72		C
73		B
74		D

/7

Subarea III (Objectives 0009–0012) Total /24

Subarea IV: Probability, Statistics, Calculus, and Discrete Mathematics

Objective 0013: Apply the principles, properties, and techniques of probability.

Question Number	Your Response	Correct Response
75		C
76		B
77		C
78		A
79		B

/5

Objective 0014: Apply the principles and concepts of descriptive statistics to the problem-solving process.

Question Number	Your Response	Correct Response
80		D
81		B
82		B
83		D
84		A

/5

Objective 0015: Apply principles and techniques of limits, continuity, and differential calculus.

Question Number	Your Response	Correct Response
85		A
86		A
87		D

Question Number	Your Response	Correct Response
88		B
89		C
90		A

/6

Objective 0016: Apply principles and techniques of integral calculus.

Question Number	Your Response	Correct Response
91		D
92		C
93		B
94		A
95		B

/5

Objective 0017: Apply the properties and techniques of discrete mathematics.

Question Number	Your Response	Correct Response
96		D
97		C
98		C
99		A
100		B

/5

Subarea IV (Objectives 0013–0017) Total /26

Practice Test Score Calculation

The practice test score calculation is provided so that you may better gauge your performance and degree of readiness to take an MTEL test at an operational administration. Although the results of this practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test, it is not possible to predict precisely how you might score on an official MTEL test.

The Sample Responses and Analyses for the open-response items may help you determine whether your responses are more similar to the strong or weak samples. The Scoring Rubric (../StudyGuide/MA_SG_CRI_63.asp#scoring) can also assist in estimating a score for your open responses. You may also wish to ask a mentor or teacher to help evaluate your responses to the open-response questions prior to calculating your total estimated score.

How to Calculate Your Practice Test Score

Review the directions in the sample below and then use the blank practice test score calculation worksheet to calculate your estimated score.

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly:	62	
Use Table 1 below to convert that number to the score and write your score in **Box A**:		**A:** 193

Open-Response Section

Enter the number of points (1 to 4) for your first open-response question:	3	
Enter the number of points (1 to 4) for your second open-response question:	3	
Add those two numbers (Number of open-response question points):	6	
Use Table 2 below to convert that number to the score and write your score in **Box B**:		**B:** 52

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score:	**A + B =** 245

Practice Test Score Calculation Worksheet: Reading Specialist (62)

Table 1:

Number of Multiple-Choice Questions Correct	Estimated MTEL Score
0 to 25	143
26 to 30	149
31 to 35	155
36 to 40	162
41 to 45	168
46 to 50	174
51 to 55	181
56 to 60	187
61 to 65	193
66 to 70	200
71 to 75	206
76 to 80	212
81 to 85	219
86 to 90	225
91 to 95	231
96 to 100	237

Table 2:

Number of Open-Response Question Points	Estimated MTEL Score
2	36

3	40
4	44
5	48
6	52
7	56
8	60

Use the form below to calculate your estimated practice test score.

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly: ☐

Use Table 1 above to convert that number to the score and write your score in **Box A**: **A:** ☐

Open-Response Section

Enter the number of points (1 to 4) for your first open-response question: ☐

Enter the number of points (1 to 4) for your second open-response question: ☐

Add those two numbers (Number of open-response question points): ☐

Use Table 2 above to convert that number to the score and write your score in **Box B**: **B:** ☐

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score: **A + B =** ☐

Evaluation Systems, Pearson, 300 Venture Way, Hadley, MA 01035